Experimental Design –Day 2

Experiment

Graphics – Exploratory Data Analysis

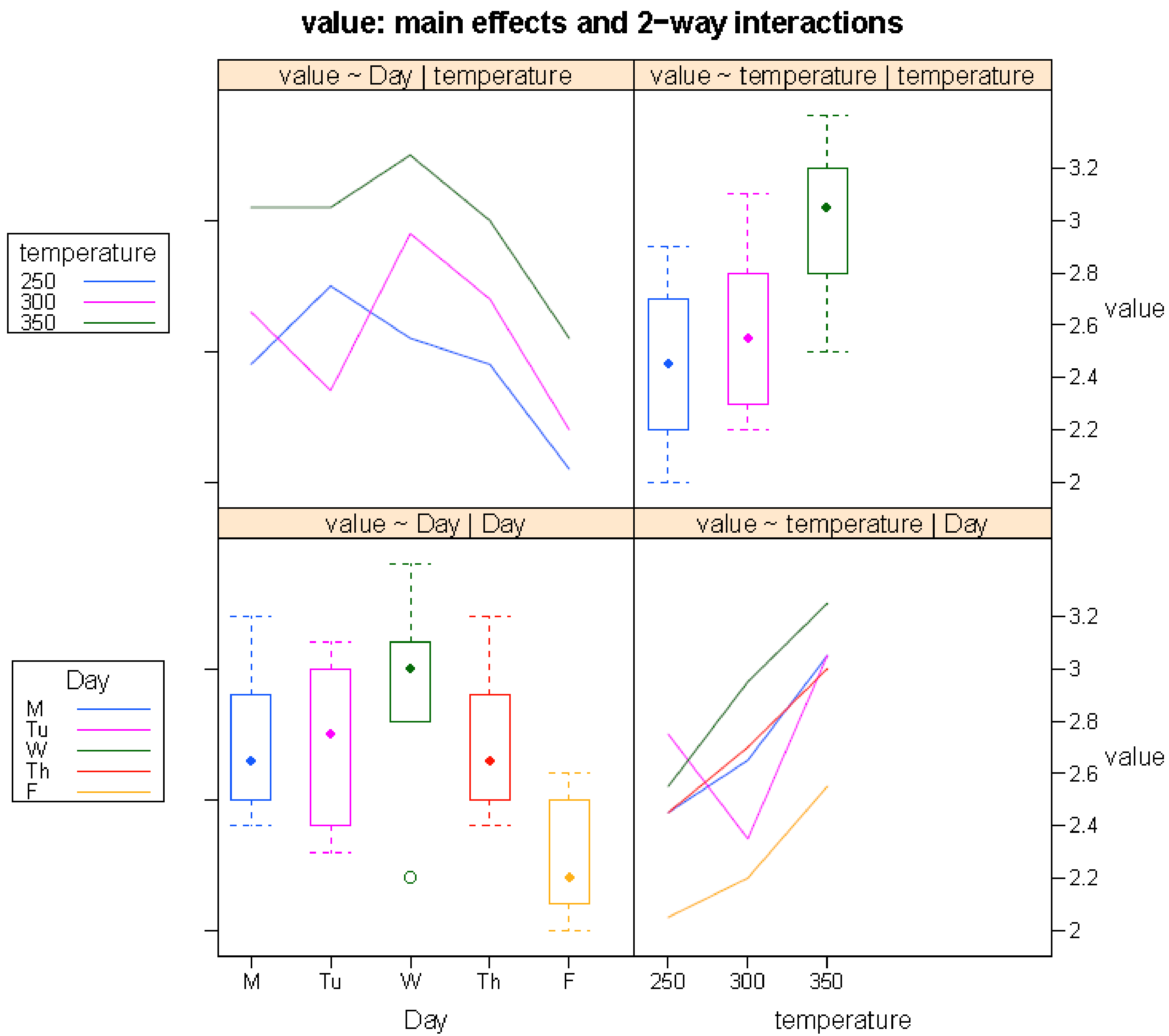
Final analytic approach

Experiments with a Single Factor

- Example: Determine the effects of temperature on process yields
  - Case I: Two levels of temperature setting
  - Case II: Three levels of temperature setting

Temperature Vs Process yields

		Temperature		
	Day	250 °F	300 °F	
Week # 1	M	2.4	2.6	Week #3
	Tu	2.7	2.4	
	W	2.2	2.8	
	Th	2.5	2.5	
	F	2.0	2.2	
Week # 2	M	2.5	2.7	Week # 4
	Tu	2.8	2.3	
	W	2.9	3.1	
	Th	2.4	2.9	
	F	2.1	2.2	


value: main effects and 2−way interactions
value ~ Day | temperature
value ~ temperature | temperature
value ~ Day | Day
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temperature
250
300
350
Day
M
Tu
W
Th
F
3.2
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value
M
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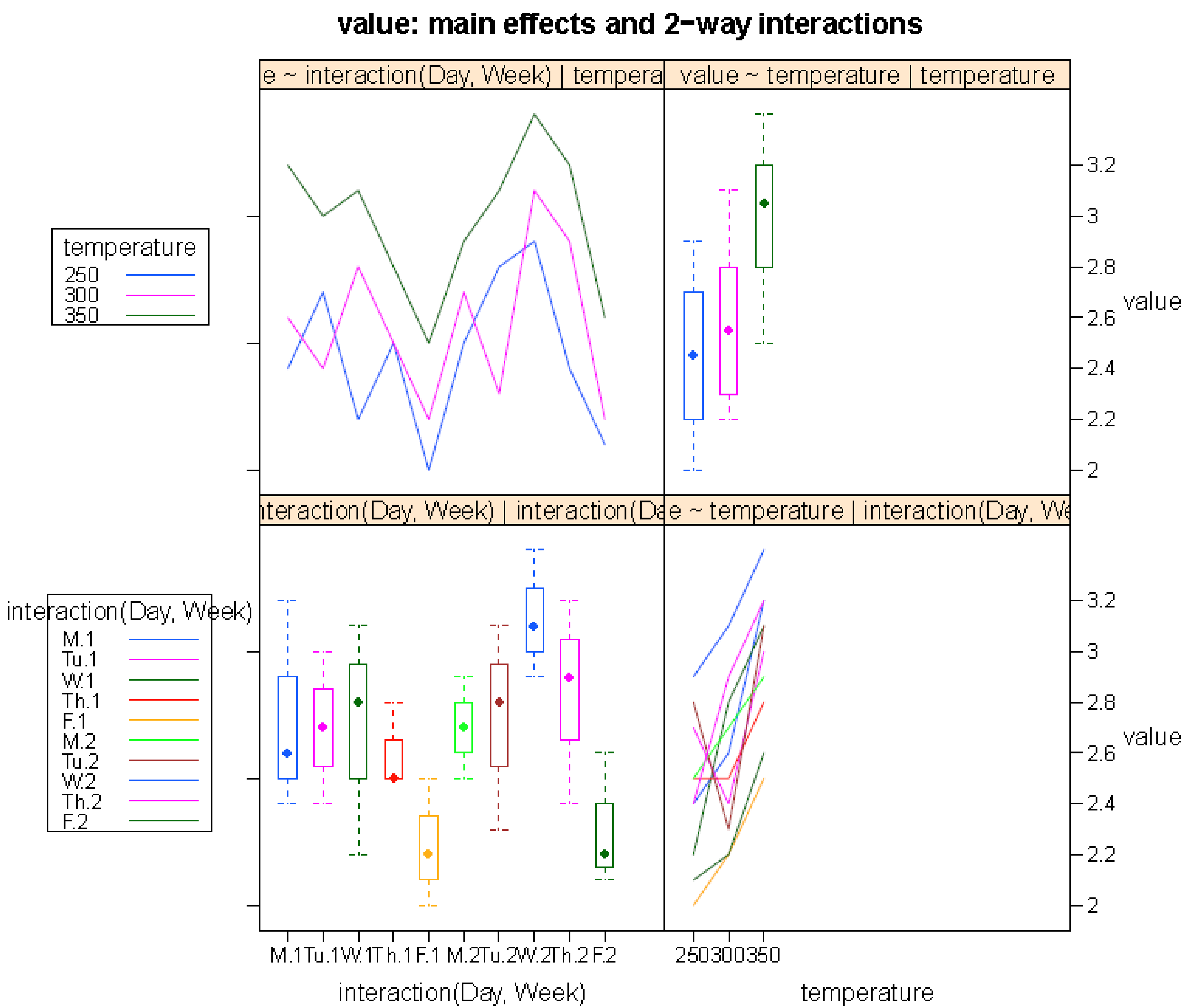
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interaction(Day, Week)
M.1
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W.1
Th.1
F.1
M.2
Tu.2
W.2
Th.2
F.2
3.2
3
2.8
2.6
2.4
2.2
2
value
M.1 Tu.1 W.1 Th.1 F.1 M.2 Tu.2 W.2 Th.2 F.2
interaction(Day, Week)
250 300 350
temperature

ANOVA for Temperature Data(3 levels)

Source of Variations	d.f.	SS	MS	F
Temperature	2	1.545	0.7725	8.91
Within	27	2.342	0.0867	**p-value=.001** **Reject H_0**
Total	29	3.887		

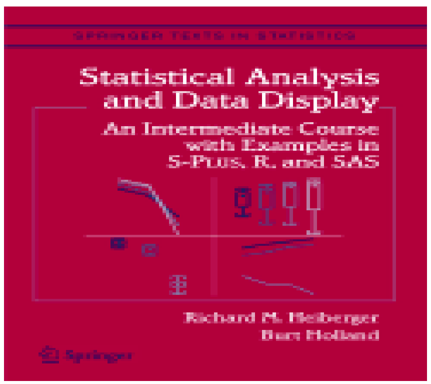
$$SS_{temp} = \sum_{i=1}^{3} \frac{\left(\sum_{j=1}^{10} y_{ij}\right)^2}{n} - \frac{\left(\sum_{i=1}^{3}\sum_{j=1}^{10} y_{ij}\right)^2}{an}$$

$$SS_{total} = \sum_{i=1}^{3}\sum_{j=1}^{10} {y_{ij}}^2 - \left(\sum_{i=1}^{3}\sum_{j=1}^{10} y_{ij}\right)^2/an$$

$SS_{within}=SS_{total}-SS_{temp}$

The figures included with this workshop have been prepared by Professor Richard Heiberger (astro.ocis.temple.edu/~rmh) in R using the **HH** package and functions described in his books.

www.R-project.org



http://www.springer.com/978-0-387-40270-3

http://www.springer.com/978-1-4419-0051-7

Randomized Block Design
Latin Square Designs
Balanced Incomplete Block Design

RBD Analysis

- This design strategy improves the accuracy of comparisons among treatments by eliminating a source of variability.
- Suppose we have, in general, *a* treatments to be compared, and *b* blocks.
- There is one observation per treatment in each block and treatments are run in random order within each block.
- The blocks represent a *restriction* on randomization.

Statistical Analysis

- We partition the total sum of squares:

$$SS_{total} = SS_{treatment} + SS_{blocks} + SS_{within}$$

- There are N total observations, so SS_{total} has N -1 degrees of freedom.
- There are a levels of the factor, so $SS_{treatment}$ has a - 1 degrees of freedom.
- There are b blocks, so SS_{blocks} has b - 1 degrees of freedom.
- Thus, we have $(a – 1)(b – 1)$ degrees of freedom for SS_{within}

ANOVA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F statistic
Treatments	$Ss_{treatment}$	a - 1	$MS_{treatment}$	$F = MS_{treatment} / MS_{within}$
Blocks	SS_{blocks}	b - 1	MS_{blocks}	$F=MS_{blocks}/MS_{within}$
Within treatments (Error)	SS_{within}	(a – 1)(b – 1)	MS_{within}	
Total	SS_{total}	N - 1		

Incorrect Analysis

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F statistic
Treatments	$SS_{treatment}$	a - 1	$MS_{treatment}$	F =
Within treatments (Error)	SS_{within}	N - a	MS_{within}	
Blocks	SS_{blocks}	b - 1	MS_{blocks}	$F=MS_{blocks}/MS_{within}$
Within treatments (Error)	SS_{within}	$(a – 1)(b – 1)$	MS_{within}	
Total	SS_{total}	N - 1		

The randomized block design reduces the amount of noise (variability) in the data sufficiently for differences among the four treatments to be detected.

Example – Metal coupons

- Determine whether or not 4 metal coupon tips produce different readings on a hardness testing machine:
  - Press the tip into a metal test coupon and measure the hardness of the coupon
  - Collect 4 obs/tip
  - One-factor ANOVA? Then: each of the 4x4 runs of the tips is assigned to one experimental unit, that is, a metal coupon. Therefore need 16 coupons, one coupon/run. Problem with variability btw coupons?

Example

- continued -

- If No variability btw coupons: eliminate that source: create blocks.
  - Each block (coupon) contains ALL tips; within a block the tips are random

Note:

- In general, Blocks = batches , people, time, units of test equipment or machinery

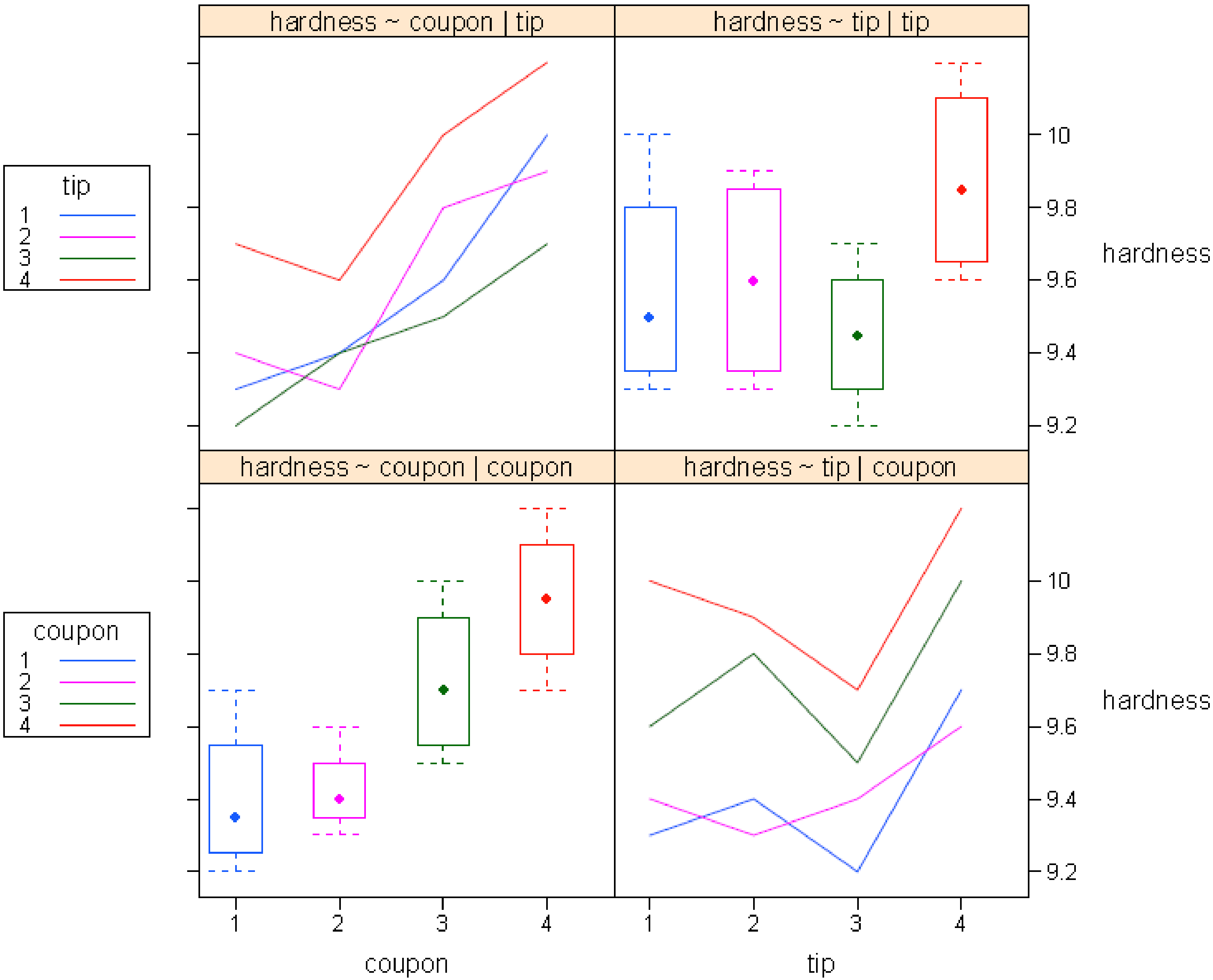
Example
- continued -

Randomized Complete Block Design for the Hardness Testing Experiment

	Test Coupon			
Type of Tip	1	2	3	4
1	9.3	9.6	9.6	10.0
2	9.4	9.3	9.8	9.9
3	9.2	9.4	9.5	9.7
4	9.7	9.6	10.0	10.2

Use Minitab for ANOVA

coupon is a block, we anticipate visible differences
tip is the treatment, we are investigating whether there are differences



Coded Data for the Metal coupons Experiment
(using Minitab)

	Coupon (Block)				
Type of Tip	**1**	**2**	**3**	**4**	$y_{i.}$
1	**-2**	**-1**	**1**	**5**	**3**
2	**-1**	**-2**	**3**	**4**	**4**
3	**-3**	**-1**	**0**	**2**	**-2**
4	**2**	**1**	**5**	**7**	**15**
$y_{.j}$	**-4**	**-3**	**9**	**18**	**20=** $y_{..}$

ANOVA
for the Metal coupons Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Treatments					
(Type of Tip)	38.50	3	12.83	14.44	0.0009
Blocks (coupons)	82.50	3	27.50		
Error	8.00	9	0.89		
Total	129.00	15			

Randomized Block Design

- It is interesting to observe the results we would have obtained had we not been aware of randomized block designs.
- Suppose we used only 4 specimens, randomly assigned the tips to each and (by chance) the same design resulted.
- The incorrect analysis of the data as a completely randomized design gives $F = 1.7$, the hypothesis of equal means cannot be rejected.

Incorrect Analysis of the Metal Coupons Experiment as a Completely Randomized Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Type of Tip	38.50	3	12.83	1.70
Error	90.50	12	7.54	
Total	129.00	15		

The randomized block design reduces the amount of noise (variability) in the data sufficiently for differences among the four treatments to be detected.

Additional Facts

- We are assuming that there is no interaction between treatments and blocks.
- If interaction is present, it can seriously affect and possibly invalidate the analysis of variance.
- In situations where both factors, as well as their possible interaction are of interest, *factorial designs* must be used, and we must have replications.

The Latin Square Design

- The randomized block design is a design that reduces the residual error in an experiment by removing the variability due to a known and controllable nuisance variable.
- There are other types of designs that utilize this blocking principle.
- The Latin square design is used to eliminate *two* nuisance sources of variability; that is, it systematically allows blocking in two directions.

Example

- A drug manufacturer is studying the effect of five different drug formulations.

  - Each formulation is mixed from a batch of raw material only large enough for five formulations to be tested.

  - The formulations are prepared by several operators with substantial differences in skills and experience.

- There are two nuisance factors to be averaged out: batches of raw material and operators.

- Solution: Use a Latin square design

Latin square designs

- The rows and columns in a Latin square design represent two restrictions on randomization.
- In general, a Latin square for p factors, or a $p \times p$ Latin square, is a square containing p rows and p columns.
- Each of the resulting squares contains one letter corresponding to a treatment, and each letter occurs once and only once in each row and column.

- Latin Square Design for 3 treatments

columns

rows

A	B	C
C	A	B
B	C	A

- Latin Square Design for 4 treatments

columns

rows

A	B	C	D
D	A	B	C
B	C	D	A
C	D	A	B

Example: Dynamite Formulation

- An experimenter is studying the effect of five different formulations of an explosive mixture
- A batch of raw material is large enough for only five formulations
- Formulations are prepared by five operators

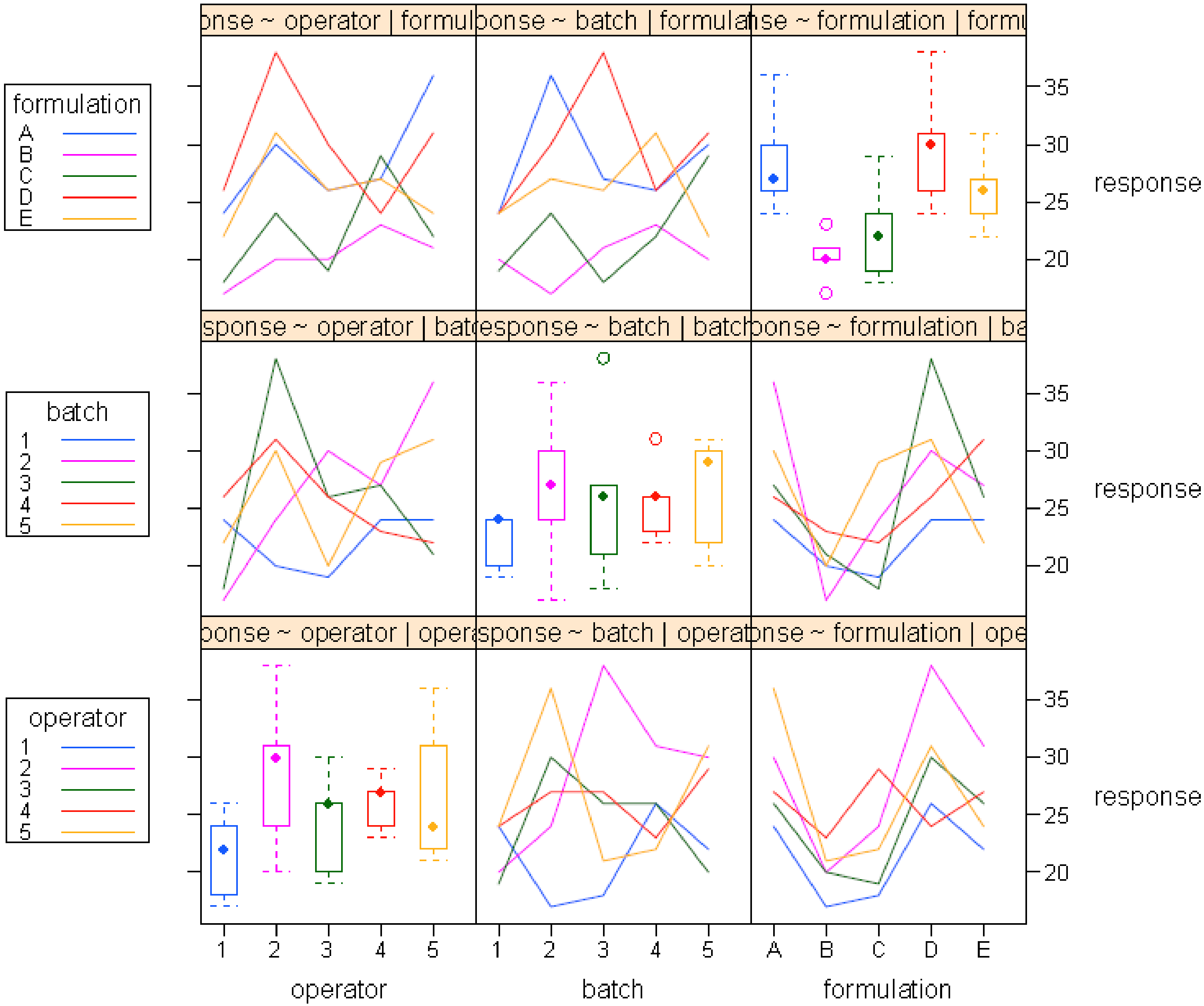
Data
for the Dynamite Formulation Example

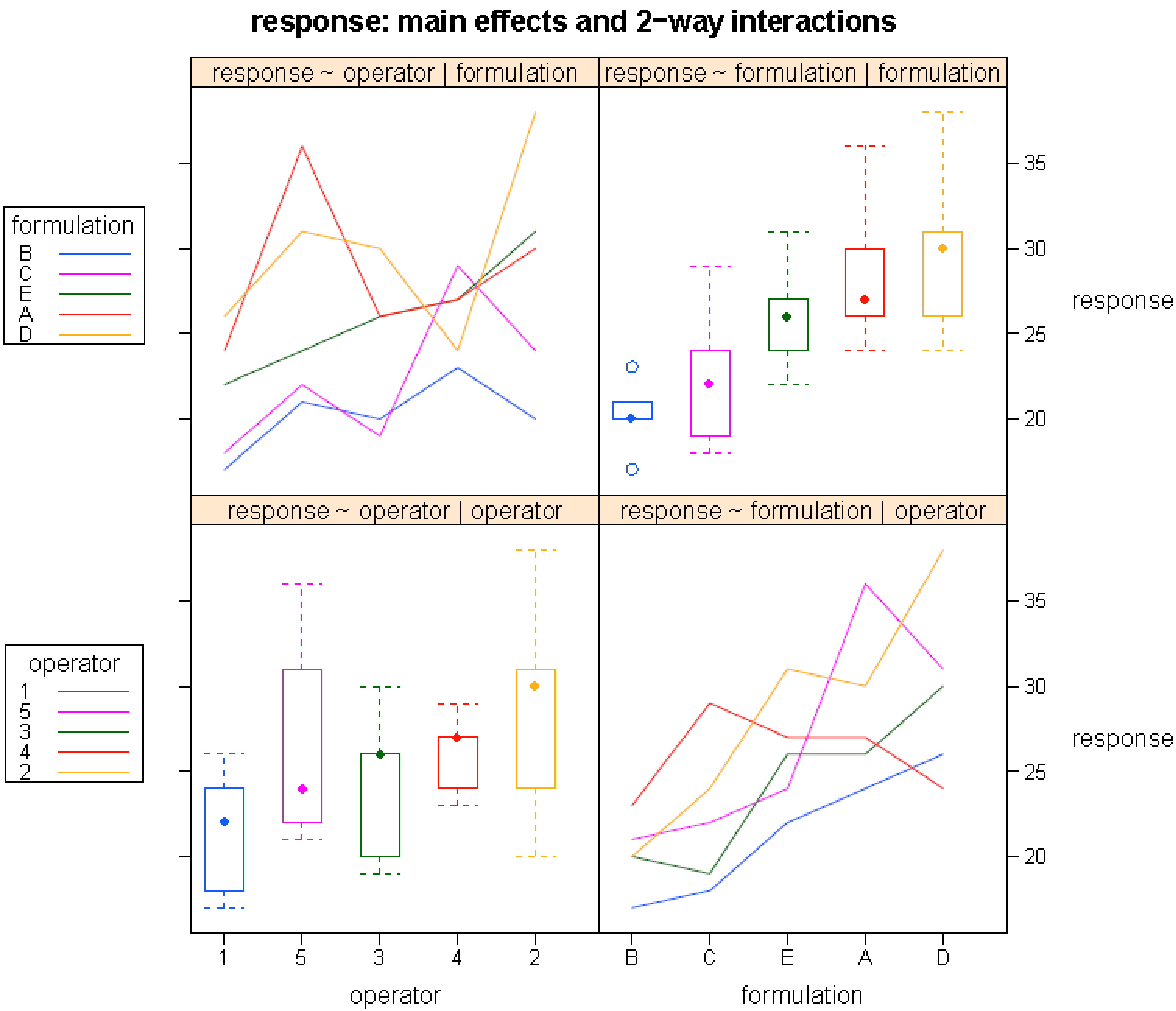
Latin Square Design

Batches of Raw Material	Operators				
	1	2	3	4	5
1	A=24	B=20	C=19	D=24	E=24
2	B=17	C=24	D=30	E=27	A=36
3	C=18	D=38	E=26	A=27	B=21
4	D=26	E=31	A=26	B=23	C=22
5	E=22	A=30	B=20	C=29	D=31

This is called **Standard Latin Square Design**: the first row is in alphabetical order

response: main effects and 2−way interactions





response: main effects and 2−way interactions
response ~ operator | formulation
response ~ formulation | formulation
response ~ operator | operator
response ~ formulation | operator
formulation
B
C
E
A
D
operator
1
5
3
4
2
35
30
25
20
response
response
1 5 3 4 2
operator
B C E A D
formulation

Coded Data for the Dynamite Formulation Example

Batches of Raw Material	Operators					$y_{i..}$
	1	2	3	4	5	
1	A= -1	B= -5	C= -6	D= -1	E= -1	-14
2	B= -8	C= -1	D= 5	E= 2	A=11	9
3	C= -7	D=13	E= 1	A= 2	B= -4	5
4	D= 1	E= 6	A= 1	B= -2	C= -3	3
5	E= -3	A= 5	B= -5	C= 4	D= 6	7
$y_{..k}$	-18	18	-4	5	9	10

Analysis of Variance for the Dynamite Formulation Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Formulations	**330.00**	**4**	**82.50**	**7.73**	**0.0025**
Batches of raw material	**68.00**	**4**	**17.00**		
Operators	**150.00**	**4**	**37.50**		
Error	**128.00**	**12**	**10.67**		
Total	**676.00**	**24**			

DF for Error: 5*5 – 4 -4-4-1 = 12 For p=5*5 LSD

Notes

- One can use replicates of the LSD (n), to increase the accuracy of error estimation
  - For 3*3 LSD there are only **2** df for error estimation;
  - For 4*4 LSD there are only **6** df for error estimation
  - For 5*5 LSD with N=5 replicates : df=88
- Could use the same operators and batches in each replicate , or could use different ones

Standard Latin Squares and Number of Latin Squares of Various Sites

Size	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	p × p
Examples of standard squares	*A B C* *B C A* *C A B*	*A B C D* *B C D A* *C D A B* *D A B C*	*A B C D E* *B A E C D* *C D A E B* *D E B A C* *E C D B A*	*A B C D E F* *B C F A D E* *C F B E A D* *D E A B F C* *E A D F C B* *F D E C B A*	*A B C D E F G* *B C D E F G A* *C D E F G A B* *D E F G A B C* *E F G A B C D* *F G A B C D E* *G A B C D E F*	*ABC . . . P* *BCD . . . A* *CDE . . . B* ⋮ *PAB . . .* (*P* − 1)
Number of standard squares	1	4	56	9408	16,942,080	—
Total number of Latin squares	12	576	161,280	818,851,200	61,479,419,904,000	$p!(p-1)! \times$ (number of standard squares)

[a] Some of the information in this table is found in *Statistical Tables for Biological, Agricultural and Medical Research*, 4th edition, by R. A. Fisher and F. Yates, Oliver and Boyd, Edinburgh, 1953. Little is known about the properties of Latin squares larger than 7 × 7.

BALANCED INCOMPLETE BLOCK DESIGN

- In certain experiments using randomized block designs, we may not be able to run all the treatment combinations in each block.
- It is possible to use randomized block designs in which every treatment is not present in every block.
- Symmetric design: any pair of treatments occur together the same number of times as any other pair.
- Could run replicates for better error estimate

Example

- Time of reaction for a chemical process is a function of the type of catalyst employed.
  - Symmetric design

Treatment (Catalyst)	Block (Batch of Raw Material)				$y_{i\cdot}$
	1	2	3	4	
A->1	73	74	--	71	218
B->2	--	75	67	72	214
C->3	73	75	68	--	216
D->4	75	--	72	75	222
$y_{\cdot i}$	221	224	207	218	$870=y_{\cdot\cdot}$

Analysis of Variance Incomplete Block Design

Source of Variation	Sum of Squares	Degree of Freedom	Mean Square	F_0
Treatments(adjusted for blocks)	22.75	3	7.58	11.66
Blocks	55.00	3	--	
Errors	3.25	5	0.65	
Total	81.00	11		

Conclusion: $11.66 > F_{0.05,3,5} = 5.41$

The catalyst employed has a significant effect on The time of reaction.