

Additional Concepts in Factorial Designs

The 3^k Factorial Design

- The 3^k Factorial Design is a factorial arrangement with k factors each at three levels.
- We refer to the three levels of the factors as low (0), intermediate (1), and high (2).
- For example, in a 3^2 design, the nine treatment combinations are denoted by 00, 01, 10, 02, 20, 11, 12, 21, 22.

The 3^k Factorial Design

- The 3^k factorial design is considered by experimenters who are concerned about curvature in the response.
- The addition of a third level allows the relationship between the response and each factor to be modeled with a quadratic relationship.
- Other alternatives:
 - response surface designs
 - 2^k design augmented with center points

Example: 3^2 Design

- The simplest 3^k factorial design is the 3^2 design, which has two factors, each at three levels.
- The $3^2 = 9$ treatment combinations are: 00, 01, 10, 02, 20, 11, 12, 21, 22.
- There are eight degrees of freedom between these nine treatment combinations: the main effects A and B have 2 degrees of freedom each, and the AB interaction has 4 degrees of freedom.

Example: 3^2 Design

- When a factor has three levels, it will have two degrees of freedom.
- Therefore, the associated sums of squares can be broken down into two components: one that represents the linear effect (SSA_L) and the other that represents the quadratic effect (SSA_Q).
- A linear effect is where the value of the response variable changes at almost a constant rate over the different levels.
- A quadratic effect is where the value of the response variable changes along the lines of a quadratic relationship.