It's RILA Time: An Introduction to Registered Index-Linked Annuities

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Abstract

Registered index-linked annuities (RILAs) are increasingly popular equity-based retirement savings products offered by U.S. life insurance companies. They combine features of fixed-index annuities and traditional variable annuities (TVAs), offering investors equity exposure with downside protection in a tax-deferred setting. This article introduces RILAs to the academic literature by describing the products' key features, developing a general pricing model, and deriving the providers' hedging strategy by decomposing their liabilities into short-term European options.

Numerical illustrations show that RILAs offer investors similar risk profiles (in the long run) as TVAs with maturity guarantees, and that many products currently sold appear to be priced quite favorably for investors. For providers, RILAs may be a preferable alternative or complement to TVAs as they greatly simplify the management of the embedded equity risk and can naturally reduce the TVA capital requirements. These features position RILAs as a viable long-term solution for this product space.

Key Words: Registered Index-Linked Annuities, Variable Annuities, Fixed-Index Annuities, Structured Annuities, Hedging.

JEL Codes: G13, G22, G52.

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1 Introduction

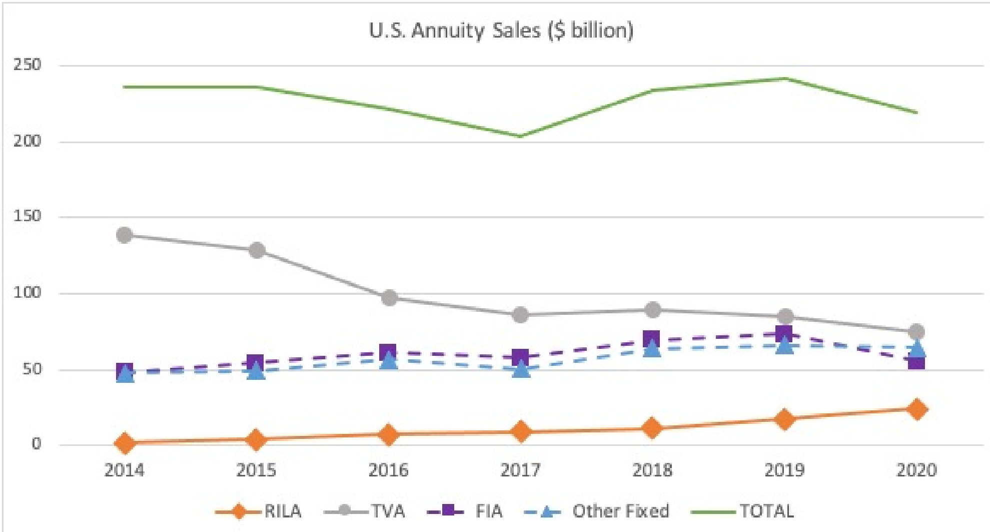
Registered index-linked annuities (RILAs)—also known as “index-linked annuities”, “structured annuities”, “structured variable annuities” or “buffer annuities”—are a relatively recent type of deferred equity-linked annuity. They can be purchased from an increasing number of life insurance companies in the U.S. and enjoy preferential tax treatment akin to other long-term retirement savings vehicles. Under a RILA contract, the account value is credited a return based on the performance of a common market index over a given term of usually one year (and longer in some cases), subject to minimum return guarantees in the form of a floor or buffer for downside protection in exchange for a cap on potential gains or a limited participation in the underlying index. This is structurally similar to *fixed-index annuities* (FIAs)—also known, especially in the academic literature, as *equity-indexed annuities* (EIAs) or, simply, *indexed annuities* (Alexandrova et al., 2017)—except that RILAs are regulated as securities since they offer increased equity exposure and may credit negative returns. By retaining some of the downside risk, investors are able to benefit from relatively generous rates on the upside.

RILAs were first offered in 2010 by AXA Equitable Life (AM Best, 2019). In 2014, they reported meager sales of just \$1.9 billion, or only around 1.4% of total variable annuity sales that year. Since then, RILAs have gained rapidly in popularity, contrary to the overall trend in the individual annuity market (see Figure 1). By 2017, RILA sales had increased nearly five-fold to around \$9.2 billion. And in 2020, U.S. insurers sold \$24.0 billion in RILA policies. This corresponds to nearly a quarter of all variable annuity sales and 11% of the entire individual annuity market. That is, in the span of only six years, RILAs have developed from a fringe product to a major force in the market.

In light of that, this article provides a timely first academic study on RILAs.[1] In the following section, I describe the key features of this product, based on a review of the available offerings in the market to date, and compare them to TVAs and FIAs. I also provide some institutional background on RILAs and discuss their potential use in qualified retirement plans. Building on these insights, the next section develops a general pricing and hedging framework for RILAs. In particular, I decompose the carrier's liabilities from the RILA contract into several short-term European options on the underlying index. This implies that providers can near-perfectly hedge the embedded equity risk by trading the corresponding

[1] Other types of equity-linked annuities have been studied extensively, dating back to Tiong (2000) for FIAs and Milevsky and Posner (2001) for TVAs, respectively. It appears, at least at first glance, that RILAs are more straightforwardly structured and entail more flexibility and less risk exposure to the providers. Therefore, I am hopeful that less research will be needed to sufficiently understand this product and its implications for investors and insurers.

Figure 1: U.S. Annuity Sales by Category, 2014 – 2020, in $ Billions



Source: LIMRA Secure Retirement Institute.

options in the financial markets, at least for common guarantee terms of one and two years. This is particularly true since the guarantees are written on only a few popular market indexes (most commonly the S&P 500, the Russell 2000, and the MSCI EAFE) that have very liquid option markets. In contrast, the long-term guarantees embedded in TVA policies have to be hedged "manually" using instruments such as futures contracts and exchange-traded funds (since the underlying mutual funds cannot be shorted), which exposes insurers to substantial levels of hedging errors and basis risk. And FIAs often include multi-term guarantees that resemble exotic options and that cannot be hedged with commonly traded derivatives (see e.g. Lee, 2003; Lin and Tan, 2003; Boyle and Tian, 2008; Gaillardetz and Lakhmiri, 2011, among many others). Therefore, RILAs are perfectly suited for insurers who want to offer their customers equity-linked savings opportunities without themselves being exposed to excessive equity risk.

Lastly, I present two numerical illustrations. First, I estimate the implicit cost and profitability of 29 empirical RILA contracts from eight major carriers. Using the earlier decomposition of the pricing function into European options (plus constants), I find that RILA guarantees offered in the market are priced rather favorably towards the investors (i.e., their guarantees are quite generous). In fact, for contracts issued in late 2019 or early 2020, I estimate an average annual cost of only around 0.17% of the RILA investment amount. That is, year over year, investors can expect to receive 99.8% of their RILA investment back (in risk-neutral expected present value terms). This is substantially better than for TVA policies (Moenig and Zhu, 2018).

Moreover, during the Covid-19 pandemic in 2020 carriers adjusted the cap rates on their products to reflect the changes in option prices due to the lower interest rates and increased market volatility at the time. This particularly affected contracts that use a *floor* for the investor's downside protection. However, many carriers were willing to retain a significant part of the resulting increase in hedging costs and absorb a reduction in surplus of over a percentage point (per annum) on average across the floor-based RILA products in my sample. Notably, these results are independent of any model specifications or parameter assumptions since I derived the present value of the liabilities under full hedging and entirely from empirical option prices. These findings suggest that RILA products—at least in their current form—are attractive to investors due to their pricing. And while carriers are likely still able to pay their expenses and make a steady profit, their earnings will have to come primarily from their ability to invest the RILA premiums into long-term and fixed-income assets that earn them a credit risk premium. In addition, RILAs can be used to decrease the risk exposure from a carrier's TVA guarantees and thus reduce reserve requirements and capital costs.

In the second part of the numerical analysis, I study RILAs in the context of long-term savings vehicles. Specifically, I assess their payout profiles under different product specifications and in comparison to other equity-linked annuities. Using a Monte Carlo simulation, I find that RILAs are considerably riskier than FIAs and offer investors a risk profile that is comparable to standard TVA contracts with maturity guarantees, but at a potentially much lower cost. In fact, the product has similar upside potential, despite the imposed caps on annual returns.[2] And on the downside, while RILAs do not fully protect investors against financial losses in any given year, the likelihood of an overall loss is quite small in the long term.

2 Product Description

2.1 RILA Product Characteristics

A RILA is a deferred equity-linked annuity product that can be purchased in the U.S. by individual investors as part of both qualified and non-qualified retirement accounts. RILAs are intended as long-term investments and, if used as such, they enjoy the same preferential

[2]While TVAs do not limit the potential gains from the investment, they often restrict its potential equity exposure. Due to the very long-term guarantees, this is particularly necessary in times of low interest rates and high market volatility, in order to keep the embedded risks and the cost of the guarantee manageable.

tax treatment that has contributed to the popularity of TVA products over the past two decades (Moenig and Bauer, 2016). Like other deferred annuity products, a RILA has an accumulation phase followed by a payout phase (sometimes called the "distribution phase"). The latter entails various ways to annuitize the accumulated account value at a specified time. Since insurers do not guarantee the interest rate used to determine these annuity payouts, the payout phase boils down to a simple pricing/valuation problem of an immediate fixed annuity and can be considered in isolation from the accumulation phase. Moreover, after an initial 3-10 year period with surrender charges (during which free withdrawals are typically limited to 10% of the account value), the investor can usually surrender her contract at the end of any guarantee crediting term and receive the full account value.[3] As a result, many RILA contracts may never reach a meaningful "payout phase" as investors opt for a lump-sum payout.

The accumulation phase is divided into crediting terms of usually one year and sometimes two, three or six years. The evolution of the investor's account value over the crediting term is tied to the performance of a particular market index such as the S&P 500 for U.S. large cap stocks, the Russell 2000 for U.S. small cap stocks, and the MSCI EAFE for stocks from developed nations outside of North America. With some carriers investors also have access to e.g. the Euro Stoxx, iShares U.S. Real Estate, MSCI Emerging Markets, Nasdaq 100, and SPDR Gold. The S&P 500 is the standard index offered by all carriers, and many carriers offer other indexes as well, possibly with different contract specifications (i.e. a different cap rate). The investor can choose which index to use over a given crediting term (among the available options) and can typically change the allocation each term.

At the end of the term, the insurer credits the RILA account with a return that is equal to the index return subject to restrictions on the upside—through a cap (i.e., a maximum credited return) and/or reduced participation in the gains of the index—and some form of downside protection. In particular, if the index performs poorly, the credited loss is lessened by either a *floor* (i.e. a maximum loss percentage), a *buffer* (i.e. losses to the index are credited to the RILA account only above a certain threshold), or a *downside participation rate* (e.g., the loss credited to the account is only 50% of the loss of the index). A more formal definition of these contract features is provided in the following section.

Table 1 presents a sample of current RILA products from the U.S. market with their key specifications. The sample was chosen to capture the breadth of the market overall while being representative of the offerings of each individual carrier, among products with guarantee crediting terms of one or two years for which full product information was available

[3]Withdrawals *during* a crediting term are generally penalized by a downwards adjustment of the crediting rate, relative to current index performance.

Table 1: Select RILA Products from U.S. Market

#	Issuer & Product Name	Surr.	τ	φ (bps)	Downside	Dec. 31, 2019			July 1, 2020				GMDB
						P^u	C	Date	P^u	C	Date	Trend	
[1]	Allianz: Index Advantage New York VA	6	1	125	$B = 30\%$		6.25%	12/3/19		9.50%	6/2/20	↗	RoP
[2]	Allianz: Index Advantage VA	6	1	125	$B = 10\%$		15.50%	12/3/19		17.50%	6/2/20	↗	RoP
[3]	Allianz: Index Advantage VA	6	1	125	$F = 10\%$		13.00%	12/3/19		11.25%	6/2/20	↘	RoP
[4]	Allianz: Index Advantage VA	6	1	125	full		3.80%	12/3/19		3.30%	6/2/20	↘	RoP
[5]	Athene: Amplify	6	1	95	$B = 10\%$		16.25%	12/22/19	105%	inf	6/22/20	↗	RoP
[6]	Athene: Amplify	6	1	95	$F = 10\%$		14.00%	12/22/19		12.25%	6/22/20	↘	RoP
[7]	Athene: Amplify	6	2	95	$B = 10\%$	110%	inf	12/22/19	120%	inf	6/22/20	↗	RoP
[8]	Athene: Amplify	6	2	95	$F = 10\%$		25.00%	12/22/19		22.00%	6/22/20	↘	RoP
[9]	Brighthouse Financial: Shield Level Select	3	1	0	$B = 10\%$		8.00%	1/1/20		9.00%	7/1/20	↗	RoP
[10]	Brighthouse Financial: Shield Level Select	6	1	0	$B = 10\%$		11.00%	1/1/20		13.00%	7/1/20	↗	RoP
[11]	CUNA Mutual: Zone Annuity	5	1	0	$F = 10\%$		10.00%	1/10/20		10.00%	7/10/20	→	n/a
[12]	CUNA Mutual: Zone Annuity	7	1	0	full		4.00%	1/10/20		3.00%	7/10/20	↘	n/a
[13]	CUNA Mutual: Zone Annuity	10	1	0	$F = 10\%$		12.00%	1/10/20		11.50%	7/10/20	↘	n/a
[14]	Great American: Index Frontier 5	5	1	0	$F = 10\%$		9.00%	12/21/19		8.00%	6/21/20	↘	RoP
[15]	Great American: Index Frontier 7	7	1	0	full		3.25%	12/21/19		3.00%	6/21/20	↘	RoP
[16]	Great American: Index Summit 6	6	1	0	$P^d = 50\%$		9.50%	12/21/19		11.50%	6/21/20	↗	RoP
[17]	Great American: Index Summit 6	6	1	0	$P^d = 50\%$	72%	inf	12/21/19	60%	inf	6/21/20	↘	RoP
[18]	Great American: Index Summit 6	6	2	0	$P^d = 50\%$		18.00%	12/21/19		24.00%	6/21/20	↗	RoP
[19]	Great American: Index Summit 6	6	2	0	$P^d = 50\%$	77%	inf	12/21/19	75%	inf	6/21/20	↘	RoP
[20]	Lincoln: Level Advantage Indexed VA	6	1	0	$B = 10\%$		12.50%	12/15/19		16.00%	6/15/20	↗	n/a
[21]	Lincoln: Level Advantage Indexed VA	6	1	0	$B = 10\%$		12.00%	12/15/19		15.50%	6/15/20	↗	RoP
[22]	Lincoln: Level Advantage Indexed VA	6	1	0	full		2.25%	12/15/19		2.25%	6/15/20	→	RoP
[23]	RiverSource: Structured Solutions	3	1	0	$F = 10\%$		7.00%	1/21/20		6.50%	4/27/20	↘	RoP
[24]	RiverSource: Structured Solutions	3	2	0	$B = 10\%$		20.00%	1/21/20		20.00%	4/27/20	→	RoP
[25]	RiverSource: Structured Solutions	6	1	170*	$B = 10\%$		inf	1/21/20	($\varphi = 230$ bps)*		4/27/20	↘	RoP
[26]	RiverSource: Structured Solutions	6	1	0	$B = 10\%$	250%	3.40%	1/21/20		3.60%	4/27/20	↗	RoP
[27]	RiverSource: Structured Solutions	6	2	0	$B = 10\%$		23.00%	1/21/20		25.00%	4/27/20	↗	RoP
[28]	Symetra: Trek Index-Linked Annuity	6	1	0	$B = 10\%$		12.00%	12/23/19		16.00%	6/22/20	↗	RoP
[29]	Symetra: Trek Index-Linked Annuity	6	1	0	$F = 10\%$		9.25%	12/23/19		8.00%	6/22/20	↘	RoP

Notes: The table displays select RILA products from the U.S. market with their key features, including the length of the surrender charge period ("Surr.", in years), the length of the crediting term τ (in years), the annual fee rate φ (in basis points), the downside protection (through either a buffer B, a floor F, or a participation rate P^d), the upside potential (represented by participation rate P^u and/or cap C), the date at which these contract features became effective for new purchases, the "trend" in pricing between the end of 2019 and mid-2020 (with ↗ indicating a favorable change for the investor), and whether a guaranteed minimum death benefit (GMDB) is included ("RoP" = return-of-premium). All rates are based on the S&P 500 index ("SPX").

(*) Contract [25] has no upside limit ($C = \infty$) but charges a continuous fee and that fee rate varies over time (here, from 170 bps at the end of 2019 to 230 bps in mid-2020).

around December 31, 2019 and July 1, 2020. This includes eight of the nine RILA carriers in early 2020.[4] The (upside) rates shown in Table 1 are all based on the S&P 500 price index (SPX) as the underlying index. Moreover, some providers also offer longer guarantees, such as a three-year term with a 20% buffer and an 85% cap rate or a six-year term with a 10% buffer and an unlimited cap. Additionally, there are a small number of products in the market with one-year terms but where the insurer guarantees the annual crediting rates for several years (typically 3 or 6 years). Still, the most common RILA policy that all carriers offer has a one-year guarantee term with either a 10% buffer or a 10% floor, an upside participation rate of 100%, and a cap of roughly 8% for floors and 16% for buffers. However, as Table 1 shows, there are considerable differences across providers regarding both variety and pricing.

The fundamental characteristics of the crediting mechanism are the downside protection structure and rates, which remain fixed, and the upside crediting rates, which the carrier can change over time, for both new policies and renewals at the end of a crediting term. This grants insurers the flexibility to adjust their pricing to changing market conditions, a desirable feature that is not available to them with TVA policies. For instance, between January and July 2020, the volatility of U.S. equity markets roughly doubled.[5] This increased the likelihood of larger gains and losses. Due to the caps on the returns, insurers retain the (upside) risk of extreme positive outcomes. However, if the RILA contains a floor, then the insurer also retains the extreme downside risk (which tends to be more valuable than the corresponding upside risk). Therefore it is perhaps unsurprising that carriers decided to reduce the cap rates on nearly all RILA contracts that include a floor, including those that provide (FIA-style) full downside protection (see the "Trend" column in Table 1). In contrast, for contracts with a buffer, the *investor* retains the extreme downside risk. This allows the carrier to benefit from the higher market volatility, resulting in increasing cap rates (between early and mid 2020) for almost all sample RILA policies from Table 1 that include a buffer.

Moreover, unlike TVAs, most RILA products do not include any fees or charges to the policyholder (see the φ column in Table 1). Instead, providers benefit from a favorable imbalance between the guarantee's downside protection and its upside cap, the ability to invest the RILA funds into e.g. corporate bonds and earn a credit risk premium (see below), and in some cases from using a price-based index that does not account for dividend payments.

[4]Source: LIMRA Secure Retirement Institute U.S. Individual Annuities Sales Survey (Q3, 2020). The other RILA carrier, *Equitable*, did not make its historical rates available.

[5]The Chicago Board Options Exchange's CBOE Volatility Index (VIX), a popular measure of the stock market's volatility expectation—derived from options written on the S&P 500 index—was 13.78 on December 31, 2019 and 28.62 on July 1, 2020.

For instance, if an investor chooses the S&P 500 index, the credited return is based on the performance of the *SPX*, which does not factor in the dividend payments of the underlying stocks.[6] Missing out on the dividends can be viewed as an implicit cost to the policyholder, comparable in magnitude to the baseline fees of typical TVA products. However, as I find in my numerical analysis below, RILA contracts are priced much more favorably for investors than TVAs (and that analysis accounts fully for the insurer's use of a price index). And, of course, insurers can just as easily offer a total return index instead of a price index and reflect the difference in a less generous cap rate.

As I document in the final "Numerical Analysis" section, RILAs offer investors an exposure to equity risk that is comparable to standard TVA products. Moreover, many RILAs include TVA-style guarantees. As Table 1 shows, a return-of-premium death benefit guarantee (GMDB) is embedded in most RILA products (without any explicit charge). Notably, TVAs are often purchased for their ability to simultaneously protect against financial risk and longevity risk through the popular lifetime withdrawal guarantees (GLWBs). To better compete for these (potential) investors, some RILA carriers have also begun to offer GLWB riders (for a separate fee) attached to the investor's RILA account value. With more carriers entering the RILA market one would expect to see more innovation in the years to come. For instance, this could entail additional financial instruments (such as different market indexes or e.g. target volatility funds) for RILA accounts to track, more fundamental changes to the return crediting structure, or additional long-term guarantees.

2.2 RILA vs. TVA and FIA

RILAs and TVAs have much in common, including their regulatory framework, their use as equity-based retirement savings vehicles with downside guarantees, and (to a large degree) their target clientele. However, there are crucial differences between the two product types. For instance, TVAs entail a direct investment of the premium(s) in one (or several) of many available mutual funds, while for RILAs the investor can only choose among a few mainstream market indexes and the premium is not directly invested in the index.

Moreover, the financial guarantees embedded in TVA policies are long-term and are only applied when the policy terminates—either at maturity or due to the policyholder's death—or if the account value reaches 0 due to guaranteed withdrawals. This makes the TVA guarantees difficult to value and hedge due to their very long-term nature (possibly 25 years

[6] For example, if the stock prices increase by 7% on average over the guarantee crediting term, in addition to paying 2% in dividends, then the RILA account will be credited (only) 7%, even though investors in the underlying stocks would earn a 9% return. This results in a benefit to the RILA carriers, e.g. through lower option prices. This would explain why some carriers are willing to offer RILA guarantees with downside protection but no upside cap (and 100% participation).

and more), strong dependence on policyholder behavior (Kling et al., 2014; Bauer et al., 2017), and exposure to basis risk (Trottier et al., 2018; Li et al., 2021). RILAs, in contrast, are designed to avoid these problems, while still providing investors similar forms of equity exposure with downside protection (and, as I show in this study, a similar distribution of long-term outcomes). In particular, RILA guarantees are short-term and are tied to the performance of a major stock index, so that insurers can hedge the embedded liabilities near-perfectly through the financial markets. The absence of fees in most RILA products (see Table 1)—and the overall pricing that is quite favorable to investors, see below—should also make the contracts more marketable compared to TVAs which many financial advisors consider overpriced.

Furthermore, RILA guarantees appear to be much less dependent on policyholder behavior than their TVA counterparts. For instance, TVA policyholders often have strong incentives to surrender or exchange their policy when the embedded guarantee moves "out of the money" (Moenig and Zhu, 2018). RILAs provide investors with the same flexibility in accessing their money when needed, but since the guarantee is always "at the money" (at least for single-term guarantees) there is little incentive for investors to anti-select against the insurer. The incentive structure for agents and brokers—who receive a new commission when they can convince a policyholder to such an exchange—may still lead to significant lapse rates, but likely lower than for TVAs. In addition, RILA carriers can further reduce lapse incentives by offering (slightly) more generous *renewal* rates compared to the cap rates for *new* policies. In fact, my review of current market offerings suggests that many carriers are already doing that, even when contracts are still subject to surrender charges. In conclusion, surrenders and exchanges appear to be less consequential for RILA products, relative to TVAs.

A particular motivation for TVA providers to transition to RILAs is the upcoming regulatory change in U.S. GAAP, known as ASU 2018-12 or "Targeted Improvements to the Accounting for Long-Duration Contracts" (LDTI), which will require life insurers to reflect their liabilities from TVA guarantees under fair (i.e. risk-neutral) market values on their income statements (Birk and Tao, 2019). This will generally lead to an upwards correction of the TVA liabilities and diminish the profit projections of these products. RILAs, in contrast will be largely unaffected by this regulatory change because insurers typically hedge the embedded equity risk.

Lastly, RILAs are formally categorized and regulated as "variable annuities" and are pooled with TVAs for accounting and regulatory purposes. By writing both product types—as seven of the top ten RILA carriers currently do—providers can potentially reduce the reserves and risk-based capital requirements of their variable annuity portfolios. This is because RILAs can act as a natural hedge for the TVA guarantees and because the premiums from RILA policies are generally held in fixed-income securities, which helps diversify the providers' exposure to equity risk from their TVA offerings. Therefore, in the years to come life insurers may be motivated to develop RILA products and to promote them more aggressively and possibly in favor of TVAs.

And finally, RILAs also bear resemblance to FIAs. In particular, one can view an FIA as a special case of a RILA with a floor of 0%. That is, the insurer provides full protection on the index return in exchange for a low cap rate of currently between 2% and 4% (see contracts [4], [12], [15] and [22] in Table 1). That is, relative to FIAs, RILAs provide considerably more exposure to the underlying market index with respect to both upside and downside risk.

2.3 Institutional Background

The first RILA introduced to the market was "Structured Capital Strategies" by AXA Equitable (now Equitable) in October 2010. For close to three years, the product was without direct competition and amassed over $3 billion in sales. Ten years later, it is still a market leader. In 2013, Brighthouse Financial (under the umbrella of MetLife), CUNA Mutual and Allianz began offering RILA products as well (Kerr et al., 2014). At the time, the new product was often referred to as a "structured annuity", but has since been largely replaced by the industry's preferred (and more descriptive) term "registered index-linked annuity".[7] As these products became increasingly popular, other carriers joined the market: Great American Life and Lincoln Financial in 2018; Athene, RiverSource and Symetra in 2019; and Nationwide, Protective Life and Prudential in 2020. This brings the total to twelve carriers by the end of September 2020. Given the rapid growth in sales—and the challenges faced by other individual annuity products—more insurers will likely follow in the years to come.

The funds invested in a RILA contract are placed in a non-unitized separate account. As such, they are insulated from the insurer's creditors but can be invested at the firm's discretion, and the realized returns on these investments are entirely the firm's.[8] Insurers will generally invest these funds—similarly to their general account—in fixed-income securities

[7] It is possible that insurers initially tried to avoid the term "index-linked" or "indexed" due to its resemblance to FIAs, which also includes the word "index" and which may have carried over a bad reputation among some investors and financial advisors due to questionable product structures and sales practices in earlier years. Source: `https://www.investmentnews.com/the-case-for-fixed-indexed-annuities-48621`, accessed February 24, 2021.

[8] This is somewhat in contrast to TVA policies which are also held in separate accounts and thus protected from the insurer's other business; however, the funds in the (unitized) TVA sub-accounts have to be invested as the policyholder chooses and returns are credited to the account directly.

and thus earn a credit risk premium. As I show in this study, this risk premium tends to be the insurers' most significant source of profit for RILA policies. Moreover, since RILAs are intended to be long-term investment vehicles—and have some built-in barriers to early surrenders—carriers can potentially invest the contributions into long-term assets that also earn a liquidity premium. This may help explain why firms offer higher cap rates for 6-year surrender terms than for 3-year surrender terms (see e.g. contracts [9] versus [10] and [24] versus [27] in Table 1).[9]

Because they expose investors to the risk of financial loss of the principal, RILAs are considered securities rather than pure insurance products. As a result, they are regulated by the Securities and Exchange Commission (SEC) and are not protected by state guaranty funds (Insured Retirement Institute, 2020). In turn, by registering the product with the SEC, RILA providers avoid having to pay into the guaranty fund. This is in contrast to fixed annuities (including FIAs), which are only regulated at the state level and protected by state guaranty funds (up to a specific dollar amount) in case the insurer becomes insolvent and is unable to fulfill its promise to the investor.

Despite their rapid rise in past years, RILAs still (as of early 2021) face some institutional constraints that may impede the continued growth of the market. First, RILA carriers must currently register any new product using standard SEC forms for new equity offerings. These have extensive requirements for filing and disclosure that are often irrelevant to the product and to prospective RILA investors. To facilitate access to RILA products, insurance representatives introduced bills H.R.6994 and S.3795 to the U.S. Congress in May 2020, with support of the Insured Retirement Institute (IRI), the American Council of Life Insurers (ACLI), the Committee of Annuity Insurers (CAI), and the National Association of Insurance and Financial Advisors (NAIFA). The bills ask the SEC to create a special registration form for RILAs that is customized to the product's characteristics and to the information relevant to prospective investors.[10]

Second, RILAs can only be sold by agents registered with the Financial Industry Regulatory Authority (FINRA). This largely prevents insurers from using independent agents working with independent marketing organizations (IMOs), which have been instrumental for the sales of e.g. FIAs and still account for around half of FIA sales. In contrast, agents made only 7% of RILA sales in 2019 (Drinkwater, 2020). Instead, RILAs are largely sold through "the broker/dealer and wirehouse channels, where they compete for shelf space with" TVAs (AM Best, 2019). In 2019, 57% of RILA sales occurred through the indepen-

[9]Another contributing factor to this discrepancy may be the carriers' front-loaded policy acquisition expenses which insurers try to distribute internally across the contract years.

[10]Source: `https://insurancenewsnet.com/innarticle/congress-urged-to-cut-regulatory-barriers-to-index-linked-annuities`, accessed June 25, 2020.

dent broker-dealer channel and all major broker-dealers now offer at least one RILA product (Drinkwater, 2020).

And third, RILAs entail considerable acquisition expenses, at least for non-qualified investments (around 7% of the premium, though this varies across providers and distribution channels). As with TVAs (see Moenig and Zhu, 2018) these up-front expenses are paid directly by the insurer who then strives to recover them year over year through an expense loading factor embedded in the pricing model.

2.4 RILAs in Qualified Retirement Plans

The opportunity for tax deferred investment growth is widely thought of as a key advantage for TVA policies that can justify their high cost (Moenig and Bauer, 2016). However, nearly two thirds of TVA assets are invested through *qualified* retirement plans (QRPs) where they provide no additional tax benefit to the plan (Insured Retirement Institute, 2020). At the end of 2019, TVAs accounted for $1.3 trillion (6.5%) of the roughly $20 trillion invested in IRAs, 401(k) and other defined contribution (DC) plans.[11]

A potential explanation for this popularity is that individuals saving for retirement often seek equity exposure due to its increased long-term growth opportunities but at the same time want their investments (partially) protected from major market downturns. Since QRPs do not generally allow investment in financial options, the primary way to achieve equity exposure with downside protection is through a mixed portfolio of stocks and bonds. For a long time, this was an appealing combination as the negative correlation between equity and bond returns provided investors some diversification benefits. However, with correlations now negligible or even positive (see e.g. Demirovic et al., 2017, and references therein), the diversification benefit has vanished. In addition, the current low interest rate climate makes bonds less attractive to investors. In this space, TVAs with guarantees provide a meaningful investment alternative as they allow exposure to the growth potential of equity markets while providing a strictly defined protection against losses.

Based on my analysis of current product offerings, I believe that RILAs can be particularly effective in this role. Like TVAs, they offer equity exposure with downside protection and similar long-term payout profiles, but are less complicated and significantly cheaper, primarily because carriers find the embedded equity risk much easier (and more cost-effective) to manage. This should make RILAs a desirable investment option within QRPs. In fact, RILAs may even present a suitable—and largely uncontroversial—Qualified Default Investment Alternative (QDIA) for employer-sponsored DC plans.

[11]Source: `https://www.statista.com/statistics/940498/assets-retirement-plans-by-type-usa/`, accessed February 25, 2021.

For many employers, *lifecycle funds* are a popular choice as QDIA. These funds offer an equity-bond mixture that is tailored to the employee's desired retirement age, with a higher proportion of equity earlier in life and then a gradual shift towards bonds as the investor approaches retirement. RILA policies can be similarly structured to provide less protection (and thus more upside potential) when the employee is younger and gradually shift towards full (FIA-style) protection closer to the investor's retirement.

3 A General Pricing Model for RILAs

3.1 Description of RILA Contract

At time 0, the investor begins a RILA contract with maturity time T by depositing a single premium P. For $s \geq 0$, let her time-s account value be denoted by A_s. Since the RILA carrier covers all acquisition expenses, we have

$$A_0 = P\,.$$

At time T, the investor can decide whether to renew the policy, take the lump-sum payout of A_T, or convert A_T into an annuity.

During the accumulation phase, the evolution of the account depends on a stock market index with time-s value S_s. Let t denote a policy anniversary date on which a new term begins (this includes the inception of the policy at time $t = 0$). That is, at time t the insurer presents the investor at least one fully specified contract to choose from. This contract has a guarantee crediting term of length τ. Usually, τ equals one year, but some carriers offer RILA contracts with multi-year guarantee terms. For notational simplicity, I denote by R_t^S the net return of the index over the term $(t, t + \tau)$. That is:

$$R_t^S = \frac{S_{t+\tau}}{S_t} - 1\,. \tag{1}$$

Let R_t^A denote the return credited to the investor's account for this term, so that:

$$A_{t+\tau} = A_t \cdot (1 - \varphi\,\tau) \cdot (1 + R_t^A)\,, \tag{2}$$

where $\varphi \geq 0$ is the annual fee rate of the RILA policy.[12] The credited return is based on

[12]Since the account return for the term is typically not credited in full until the end of the term, the RILA policies in the market that do charge a positive fee rate base the fee on the account value *prior to* the crediting of the return. This is captured by Equation (2), as long as there are no withdrawals during the term.

the index return, but subject to either a floor F, a buffer B, or a participation rate P^d on the downside (i.e., when $R_t^S < 0$), and to a cap C and a participation rate P^u on the upside (i.e., when $R_t^S \geq 0$). That is:

$$R_t^A = \begin{cases} \min\{R_t^S + B, 0\} \cdot \mathbb{1}_{\{R_t^S < 0\}} + P^u \cdot \min\{R_t^S, C\} \cdot \mathbb{1}_{\{R_t^S \geq 0\}} & , \text{ if } B > 0 \\ \max\{R_t^S, -F\} \cdot \mathbb{1}_{\{R_t^S < 0\}} + P^u \cdot \min\{R_t^S, C\} \cdot \mathbb{1}_{\{R_t^S \geq 0\}} & , \text{ if } F < 1 \\ P^d \cdot R_t^S \cdot \mathbb{1}_{\{R_t^S < 0\}} + P^u \cdot \min\{R_t^S, C\} \cdot \mathbb{1}_{\{R_t^S \geq 0\}} & , \text{ if } P^d < 1 \, , \end{cases} \tag{3}$$

where $\mathbb{1}_{\{x\}}$ represents the indicator function with respect to statement x.[13]

3.2 Insurer's Loss

At time t, the insurer can invest the RILA account value A_t—minus expenses and hedging costs—in fixed-income assets, thus earning a credit risk premium in addition to the market interest rate. Let $r_{s,u}$ and $\alpha_{s,u}$ denote the realized annual risk-free rate and the credit risk premium, respectively, over the term from time s to time $u > s$. For notational convenience, I let

$$b_t = e^{(r_{t,t+\tau} + \alpha_{t,t+\tau}) \cdot \tau}$$

denote the accumulated value of \$1 from time t to time $t + \tau$ for this investment. For ease of exposition, I combine the insurer's expected expenses, profit margin, and other costs into an annual expense loading factor $\epsilon \geq 0$, assessed against the account value at the beginning of the term. Lastly, I also include a hedging portfolio that costs H_t to set up at time t and that pays $H_{t+\tau}$ at the end of the term.

The insurer's loss random variable over the guarantee term $(t, t + \tau)$, assessed at time $t + \tau$ and denoted by $L_{t+\tau}$, is therefore given by:

$$L_{t+\tau} = A_{t+\tau} - (A_t - A_t \, \epsilon \, \tau - H_t) \, b_t - H_{t+\tau} \, . \tag{4}$$

[13]Equation (3) presumes that the cap applies to the return of the underlying index (rather than the account itself), which becomes relevant if the RILA contract includes a finite cap $C < \infty$ as well as an upside participation rate $P^u \neq 1$. It is straightforward to adjust the formula for other crediting mechanisms.

In particular, if the RILA contains a *buffer* $B > 0$, the loss function is given by:[14]

$$\begin{aligned} L_{t+\tau}^{\text{Buffer}} &= -A_t\,(b_t - 1) + A_t\,\epsilon\,\tau\,b_t - A_t\,\varphi\,\tau - H_{t+\tau}^{\text{Buffer}} + H_t^{\text{Buffer}}\,b_t \\ &\quad - \tfrac{A_t\,(1-\varphi\,\tau)}{S_t} \cdot \text{Put}_{t+\tau}(S_t(1-B)) \\ &\quad + \tfrac{A_t\,(1-\varphi\,\tau)\,P^u}{S_t} \cdot \left(\text{Call}_{t+\tau}(S_t) - \text{Call}_{t+\tau}(S_t(1+C))\right)\ , \end{aligned} \tag{4B}$$

where $\text{Call}_s(K)$ and $\text{Put}_s(K)$ denote the payout of a European call and a European put option on the index with strike K at the time of maturity s, respectively.

And in the case of a *floor* $F > 0$:

$$\begin{aligned} L_{t+\tau}^{\text{Floor}} &= -A_t\,(b_t - 1) + A_t\,\epsilon\,\tau\,b_t - A_t\,\varphi\,\tau - H_{t+\tau}^{\text{Floor}} + H_t^{\text{Floor}}\,b_t \\ &\quad + \tfrac{A_t\,(1-\varphi\,\tau)}{S_t} \cdot \left(\text{Put}_{t+\tau}(S_t(1-F)) - \text{Put}_{t+\tau}(S_t)\right) \\ &\quad + \tfrac{A_t\,(1-\varphi\,\tau)\,P^u}{S_t} \cdot \left(\text{Call}_{t+\tau}(S_t) - \text{Call}_{t+\tau}(S_t(1+C))\right)\ , \end{aligned} \tag{4F}$$

which for $P^u = 1$ can be simplified to:

$$\begin{aligned} L_{t+\tau}^{\text{Floor}'} &= -A_t\,(b_t - 1) + A_t\,\epsilon\,\tau\,b_t - A_t\,\varphi\,\tau - H_{t+\tau}^{\text{Floor}'} + H_t^{\text{Floor}'}\,b_t - A_t\,(1-\varphi\,\tau)\,F \\ &\quad + \tfrac{A_t\,(1-\varphi\,\tau)}{S_t} \cdot \left(\text{Call}_{t+\tau}(S_t(1-F)) - \text{Call}_{t+\tau}(S_t(1+C))\right)\ . \end{aligned} \tag{4F'}$$

Lastly, if the RILA offers protection in the form of a *downside participation rate* $P^d < 1$, the insurer's loss over the term can be expressed as:

$$\begin{aligned} L_{t+\tau}^{\text{Part}} &= -A_t\,(b_t - 1) + A_t\,\epsilon\,\tau\,b_t - A_t\,\varphi\,\tau - H_{t+\tau}^{\text{Part}} + H_t^{\text{Part}}\,b_t - \tfrac{A_t\,(1-\varphi\,\tau)\,P^d}{S_t} \cdot \text{Put}_{t+\tau}(S_t) \\ &\quad + \tfrac{A_t\,(1-\varphi\,\tau)\,P^u}{S_t} \cdot \left(\text{Call}_{t+\tau}(S_t) - \text{Call}_{t+\tau}(S_t(1+C))\right)\ . \end{aligned} \tag{4P}$$

Note that in all cases, $L_{t+\tau}$ is expressed solely in terms of the contract features (B, F, P^d, P^u, C, φ, τ), the expense loading factor ϵ, and the (joint) distribution of interest rate, credit risk premium, and index return over the crediting term. Moreover, Equations (4B), (4F) and (4P) show that the insurer's loss can be broken down into (i) the interest (plus risk premium) earned on the initial account value, $A_t(b_t - 1)$; (ii) the insurer's expenses, $A_t\,\tau\,\epsilon\,b_t$; (iii) the fees collected from the account, $A_t\,\varphi\,\tau$; (iv) the payout of the guarantee in the form of a series of European options maturing at time $t+\tau$; and (v) the profit from the insurer's hedge, $H_{t+\tau}^D - H_t^D\,b_t$ for downside protection mechanism $D \in \{\text{Buffer}, \text{Floor}, \text{Floor}', \text{Part.}\}$.

[14]The proof of this and the following equations can be found in Appendix A.

3.3 Hedging RILA Guarantees

The hedging portfolio is designed to offset the insurer's option-like liabilities resulting from the return crediting mechanism. In fact, Equations (4B), (4F) and (4P) illustrate that the insurer can fully hedge its equity risk for the guarantee term by investing in the respective options. For instance, if the policy contains a *buffer*, Equations (4B) implies that the insurer should trade at time t the following three European options on the index $\{S_t\}$, all maturing at time $t+\tau$:

- Short $A_t\,(1-\varphi\,\tau)/S_t$ units of a put with strike $S_t(1-B)$.
- Long $A_t\,(1-\varphi\,\tau)\,P^u/S_t$ units of a call with strike S_t.
- Short $A_t\,(1-\varphi\,\tau)\,P^u/S_t$ units of a call with strike $S_t(1+C)$.

This hedge portfolio provides a maturity payout of

$$H_{t+\tau}^{\text{Buffer}} = -\frac{A_t\,(1-\varphi\,\tau)}{S_t}\cdot\text{Put}_{t+\tau}(S_t(1-B))+\frac{A_t\,(1-\varphi\,\tau)\,P^u}{S_t}\cdot\left(\text{Call}_{t+\tau}(S_t)-\text{Call}_{t+\tau}(S_t(1+C))\right),$$

at an initial cost of

$$H_t^{\text{Buffer}} = -\frac{A_t(1-\varphi\,\tau)}{S_t}\cdot\text{Put}_t(S_t(1-B),\tau)+\frac{A_t(1-\varphi\,\tau)P^u}{S_t}\cdot\left(\text{Call}_t(S_t,\tau)-\text{Call}_t(S_t(1+C),\tau)\right).$$

Here, $\text{Call}_s(K,u)$ and $\text{Put}_s(K,u)$ denote the time-s prices of a European call and a European put option on the index with strike K and time to maturity u, respectively.

Similarly, if the RILA account is protected by a *floor*, the insurer should trade the following four European options on the index $\{S_t\}$, all maturing at time $t+\tau$:

- Long $A_t\,(1-\varphi\,\tau)/S_t$ units of a put with strike $S_t(1-F)$.
- Short $A_t\,(1-\varphi\,\tau)/S_t$ units of a put with strike S_t.
- Long $A_t\,(1-\varphi\,\tau)\,P^u/S_t$ units of a call with strike S_t.
- Short $A_t\,(1-\varphi\,\tau)\,P^u/S_t$ units of a call with strike $S_t(1+C)$.

This hedging strategy offsets the option liabilities from the RILA contract, at an initial cost of

$$\begin{aligned} H_t^{\text{Floor}} &= \tfrac{A_t(1-\varphi\,\tau)}{S_t}\cdot\left(\text{Put}_t(S_t(1-F),\tau)-\text{Put}_t(S_t,\tau)\right) \\ &\qquad +\tfrac{A_t(1-\varphi\,\tau)P^u}{S_t}\cdot\left(\text{Call}_t(S_t,\tau)-\text{Call}_t(S_t(1+C),\tau)\right) . \end{aligned}$$

In the special case of a floor with $P^u=1$, the hedging strategy consists of:

- Borrow $A_t\,(1-\varphi\,\tau)\,F\cdot\mathbb{E}_t\,[e^{-r_{t,t+\tau}\cdot\tau}]$.
- Long $A_t\,(1-\varphi\,\tau)/S_t$ units of a call with strike $S_t(1-F)$.
- Short $A_t\,(1-\varphi\,\tau)/S_t$ units of a call with strike $S_t(1+C)$.

This costs the insurer:

$$H_t^{\text{Floor}'} = -A_t\,(1-\varphi\,\tau)\,F\cdot\mathbb{E}_t\left[e^{-r_{t,t+\tau}\cdot\tau}\right]+\frac{A_t(1-\varphi\,\tau)}{S_t}\cdot\left(\text{Call}_t(S_t(1-F),\tau)-\text{Call}_t(S_t(1+C),\tau)\right).$$

And lastly, in the case where the policyholder benefits from a *downside participation rate* below 1, the insurer can hedge its liabilities as follows:

- Short $A_t\,(1-\varphi\,\tau)\,P^d/S_t$ units of a put with strike S_t.
- Long $A_t\,(1-\varphi\,\tau)\,P^u/S_t$ units of a call with strike S_t.
- Short $A_t\,(1-\varphi\,\tau)\,P^u/S_t$ units of a call with strike $S_t(1+C)$.

The insurer's cost from this strategy is then given by:

$$H_t^{\text{Part.}} = -\frac{A_t\,(1-\varphi\,\tau)\,P^d}{S_t}\cdot\text{Put}_t(S_t,\tau)+\frac{A_t\,(1-\varphi\,\tau)\,P^u}{S_t}\cdot\left(\text{Call}_t(S_t,\tau)-\text{Call}_t(S_t(1+C),\tau)\right).$$

For the usual short terms τ of one or two years, and for common indexes such as *SPX*, such options are generally available for a variety of strike prices and maturities. Therefore, the provider will likely be able to arrange for combined option payouts that closely approximate its liabilities from the RILA policy at the end of the term. This way, the insurer is able to transfer away its entire exposure to equity risk from this guarantee at a relatively low cost.[15] By doing so, the firm's hedged loss for the term $(t, t+\tau)$ under downside protection mechanism $D \in \{\text{Buffer}, \text{Floor}, \text{Floor}', \text{Part.}\}$ is given by:

$$HL_{t+\tau}^{D} = -A_t\,(b_t-1)+A_t\,\epsilon\,\tau\,b_t-A_t\,\varphi\,\tau+H_t^D\,b_t\,. \tag{5}$$

Note that this hedged loss does not include any exposure to equity risk, but only to its investment risk which the insurer can control. This is fundamentally different from TVA products, for which the insurer remains exposed to substantial amounts of equity risk, despite its best efforts to mitigate them.

[15] For longer terms τ of e.g. 6 years, such options are not currently available in the financial markets. In that case, the insurer needs to construct its own hedging portfolio, which it can achieve with futures contracts or exchange-traded funds that aim to track the underlying index. Since the insurer likely has many RILA contracts using the same index, renewed on the same date, and electing the same guarantees—and thus the same option strikes—it can potentially benefit from economies of scale.

Finally, note that an FIA with a single-term guarantee is a special case of the above contracts, represented by $B = 1$, $F = 0$, or $P^d = 0$, respectively. In each case, the insurer's hedged loss function is given by Equation (5), with

$$H_t^{\text{FIA}} = \frac{A_t\,(1-\varphi\,\tau)\,P^u}{S_t} \cdot \left(\text{Call}_t(S_t,\tau) - \text{Call}_t(S_t(1+C),\tau)\right) \ .$$

3.4 Pricing the RILA Guarantee

Building on the above work, the insurer's pricing approach for a RILA contract with a single-term guarantee is straightforward. Under downside protection mechanism $D \in \{\text{Buffer}, \text{Floor}, \text{Floor}', \text{Part.}\}$ the firm's time-t surplus from the contract over the guarantee term $(t, t+\tau)$ is given by:

$$\begin{aligned} \Pi_t^D &= -\mathbb{E}_t\left[e^{-r_{t,t+\tau}\cdot\tau} \cdot HL_{t+\tau}^D\right] \\ &= -A_t\,(1-\varphi\,\tau) \cdot \mathbb{E}_t\left[e^{-r_{t,t+\tau}\cdot\tau}\right] + \left(A_t\,(1-\epsilon\,\tau) - H_t^D\right) \cdot \mathbb{E}_t\left[e^{\alpha_{t,t+\tau}\cdot\tau}\right] \ . \end{aligned} \tag{6}$$

As discussed in the previous section, the specifics of the downside protection are fixed over the term of the contract, and for each such specification, the insurer can set the upside rates (i.e., P^u and/or C) at time t for the upcoming crediting term. The objective of setting P^u and C should be that the surplus is nonnegative, i.e., $\Pi_t^D \geq 0$ for downside protection method D, as defined in Equation (6).

3.5 Pricing Multi-Term RILA Guarantees

A few RILA products in the market have their contracts guaranteed over multiple terms, until the maturity time T. Here, the provider will not be able to reset C and P after each term, and thus the pricing model has to take into account the return projections for the full period $(t, t+T)$. In particular, the insurer's expected surplus for downside protection method D over the full period is given by

$$\Pi_t^{D,\text{full}} = \sum_{i=1}^{T/\tau} -\mathbb{E}_t\left[e^{-r_{t,t+i\tau}\cdot i\cdot\tau} \cdot HL_{t+i\tau}^D\right] \ .$$

Notably, this guarantee type provides additional security to the investors, but also additional risks for the RILA providers. In addition, it may provide incentives for policyholder to anti-select against the providers by choosing to surrender the contract when the guarantee moves out of the money. If multi-term guarantees become more popular, this may be worth exploring in future research.

4 Numerical Analysis: RILA Pricing & Profitability

In this section, I conduct numerical analyses to assess RILA contracts regarding their pricing and profitability, based on current market offerings.

4.1 Pricing Analysis

I first consider the *pricing* of the RILA guarantees, following the model in the previous section. Specifically, I assess how attractive the contracts sold in the market were for investors using the issuer's surplus definition of Equation (6). To that effect, I study the RILA products shown in Table 1, which includes 29 distinct contracts from eight different carriers, with the S&P 500 (SPX) as the underlying index. While some carriers offer additional contract specifications and indexes, I consider the contracts shown in Table 1 sufficiently representative—for each insurer and for the market overall—among guarantees with single terms of one or two years. The latter restriction is only to ensure that the results of this pricing analysis are *model-independent*.[16]

To assess the pricing of these contracts both before and during the Covid-19 pandemic, I use the empirical prices of European options on the SPX price index on December 31, 2019 and July 1, 2020, with maturity dates December 31, 2020 and June 30, 2021 for term length $\tau = 1$ year, and maturity dates December 17, 2021 and June 17, 2022 for $\tau = 2$ years, respectively.[17] In addition, I approximate the expected discount factor $\mathbb{E}_t\left[e^{-r_{t,t+\tau}\cdot\tau}\right]$ from the 1-year and 2-year U.S. Treasury Constant Maturity Rates on December 31, 2019 (1.59% for one year and 1.58% for two years) and on July 1, 2020 (0.16% for both a one-year and a two-year term), and I denote these rates by $\hat{r}_{t,t+\tau}$.[18] That is:

$$\mathbb{E}_t\left[e^{-r_{t,t+\tau}\cdot\tau}\right] = e^{-\hat{r}_{t,t+\tau}\cdot\tau} .$$

Since pricing is typically done under the risk-neutral probability measure, I set both

[16] Several providers offer longer crediting terms (typically 3 years and 6 years), but there are no financial options available in the market with those maturities. Therefore, valuing these contracts requires a separate option pricing model and the results may vary greatly based on model specifications. In contrast, for the shorter terms I can directly use the market prices of the respective options at the corresponding times. Moreover, I only consider single-term guarantees, since there are very few RILA contracts with multi-term guarantees and since those would be more difficult to project due to the embedded uncertainty about future market movements, policyholder behavior, etc.

[17] Source: OptionMetrics, IvyDB Global Indices and `www.barchart.com`. Option prices are from market closing, using "midpoint" prices (i.e. halfway between the bid price and the ask price). Results are similar when using the bid prices or the ask prices, respectively. Since options are traded only for select strike prices, I interpolate linearly over the strikes. The value of the SPX index at the end of December 31, 2019 was 3,230.78, and at market closing on July 1, 2020 the index was at 3,115.86.

[18] Source: Federal Reserve Economic Data (FRED), *Federal Reserve Bank of St. Louis.*

the issuer's expense rate ϵ and the credit risk premium $\alpha_{t,t+\tau}$ to zero. Since all equity risk has been eliminated through the appropriate hedging strategy (using empirical, i.e. market-consistent, option prices), no further adjustments are needed. As a result, following Equation (6), the annual (risk-neutral expected) cost (or drawdown) of the RILA contract to the investor is given by:

$$C_t^D = \frac{1}{\tau} \cdot \left(-A_t \left(1 - \varphi\, \tau\right) \cdot e^{-\hat{r}_{t,t+\tau} \cdot \tau} + A_t - H_t^D \right) \,. \tag{7}$$

For instance, with an initial investment of $A_t = 100$ a value of $C_t^D = 1.0$ implies that the investor gets 99% of her investment back (in risk-neutral expected present value terms). Note that the RILA investment is held in a separate account that is protected from the insurer's creditors. As a result, the investor has no exposure to credit risk. Therefore, C_t^D is reflective of the value of the RILA product to the investor.

For each sample contract described in Table 1, the estimated annual hedging costs H_t/τ and cost/drawdown C_t^D are shown in Table 2 for the two valuation dates. I find that the hedging costs vary greatly, both across the 29 contracts and over time. In late 2019, carriers had to spend on average 1.75% of the account value per contract year in order to hedge the equity risk embedded in the RILA contracts. The aforementioned changes to the cap rates in response to the increased market volatility actually reduced the net hedging cost to around 1.0% of the account value (per year) in mid-2020. This is largely due to a reduction in net hedging costs for buffer contracts, since carriers benefitted from the increased prices of in-the-money call and put options which they shorted as part of their hedging strategy.

The estimated cost C_t also shows some variety across the product space and is in fact *negative* for many of these contracts, especially in mid-2020. And even when the cost is positive, it is rarely above 0.5% p.a. under market-consistent valuation. On average, the cost or drawdown of a RILA policy to the investor was only around 17 basis points before and negative 44 basis points during the pandemic. These findings suggest that even in the long run RILAs are substantially cheaper than TVA policies which pay back investors (in present value terms) only up to 85% of their contributions (Moenig and Zhu, 2018).

4.2 Profitability Analysis

Importantly, the previous analysis does not imply that RILA carriers are losing money on these products,[19] but rather that their surplus would have to come almost entirely from the credit spread earned from investing the RILA account balance in fixed-income securities.

[19]To the contrary: the actuaries I spoke with all emphasized that RILAs generate a relatively consistent surplus for them.

Table 2: Pricing & Profitability of RILA Sample Contracts

Contract	τ	Dec. 31, 2019			July 1, 2020		
		H_t/τ	C_t	Π_t^{gross}	H_t/τ	C_t	Π_t^{gross}
[1]	1	2.56	0.2402	1.9605	2.33	−0.9244	0.7998
[2]	1	2.19	0.6038	2.3305	0.52	0.8890	2.6452
[3]	1	2.41	0.3900	2.1130	2.23	−0.8219	0.9042
[4]	1	2.13	0.6612	2.3889	1.89	−0.4824	1.2496
[5]	1	2.26	0.2383	3.0538	2.41	−1.2985	1.5128
[6]	1	2.55	−0.0473	2.7600	2.58	−1.4755	1.3308
[7]	2	1.98	0.4797	3.2859	1.76	−0.6548	2.1645
[8]	2	2.32	0.1401	2.9265	2.32	−1.2168	1.5696
[9]	1	0.65	0.9170	3.2185	−2.04	2.1975	4.5612
[10]	1	1.53	0.0307	2.3116	−0.56	0.7178	3.0472
[11]	1	1.79	−0.2241	2.1414	1.78	−1.6236	0.7420
[12]	1	2.26	−0.6970	1.6570	1.75	−1.5881	0.7783
[13]	1	2.26	−0.6921	1.6621	2.35	−2.1901	0.1618
[14]	1	1.49	0.0713	2.6660	0.93	−0.7706	1.8390
[15]	1	1.87	−0.3094	2.2754	1.75	−1.5881	0.9999
[16]	1	1.41	0.1541	2.7510	0.66	−0.4972	2.1195
[17]	1	1.31	0.2546	2.8541	0.20	−0.0446	2.5842
[18]	2	1.30	0.2393	2.8385	1.09	−0.9337	1.6768
[19]	2	1.17	0.3765	2.9830	0.77	−0.6099	2.0178
[20]	1	1.84	−0.2776	1.8456	0.23	−0.0723	2.0857
[21]	1	1.75	−0.1850	1.9402	0.13	0.0279	2.1882
[22]	1	1.33	0.2340	2.3682	1.33	−1.1654	0.9690
[23]	1	0.78	0.7867	3.1054	0.23	−0.0655	2.2662
[24]	2	0.87	0.6731	2.9962	0.68	−0.5194	1.8127
[25]	1	2.70	0.5394	2.8132	1.95	0.5099	2.8013
[26]	1	1.53	0.0363	2.3374	−1.57	1.7281	4.1017
[27]	2	1.08	0.4645	2.7777	−0.30	0.4590	2.8373
[28]	1	1.75	−0.1850	1.6195	0.23	−0.0723	1.7601
[29]	1	1.57	−0.0061	1.8017	0.93	−0.7706	1.0490
Average:							
Overall		1.75	0.1692	2.4753	0.99	−0.4433	1.8819
Buffer only		1.75	0.2750	2.4993	0.44	0.2298	2.4860
Floor only		1.90	0.0256	2.3221	1.67	−1.1466	1.1550

Notes: For each contract [1] to [29], the table displays the issuer's estimated annual cost H_t/τ of hedging the embedded guarantees as well as the RILA's annual cost/drawdown C_t and the issuer's annual gross surplus Π_t^{gross} over the guarantee crediting term (of length τ). The three quantities are defined in the Model" section of the paper and in Equations (7) and (8), respectively. The values shown in this table are based on a nominal initial account value of 100 and empirical prices of European call and put options on the *SPX* index on December 31, 2019 and on July 1, 2020 (close of business). Details on the 29 contracts can be found in Table 1.

To assess the contracts' profitability more closely, I compute the carriers' (annualized) *gross surplus* Π_t^{gross}, again following Equation (6). The difference to the pricing analysis in the previous subsection is that the profitability analysis is carried out under the *real-world* probability measure. Notably, the embedded equity risk is fully hedged. Moreover, I focus on measuring profitability *before* expenses—that is, I set $\epsilon = 0$—as the magnitude of expenses is difficult to estimate and likely varies across providers.[20] The carrier's annualized expected gross surplus is then given by

$$\Pi_t^{\text{gross},D} = \frac{1}{\tau} \cdot \left(-A_t \left(1 - \varphi\, \tau\right) \cdot e^{-\hat{r}_{t,t+\tau} \cdot \tau} + \left(A_t - H_t^D\right) \cdot e^{\hat{\alpha} \cdot \tau} \right) \,, \tag{8}$$

where $\hat{\alpha}$ denotes the carrier's expected (annual) risk premium earned over the crediting term. Since the carrier can invest the RILA funds at its discretion it is likely to pursue investment strategies similar to its general account. I therefore proxy this rate by using each carrier's 2019 realized excess return, based on the "net yield on invested assets" as reported in the firm's financial statements,[21] minus the realized risk-free rate of return over the year 2019.[22]

Table 2 displays the gross surplus estimates of the 29 sample contracts at the two sample times. Judging by these results, RILAs appear to be sufficiently profitable to cover the associated expenses and other costs (such as the return-of-premium death benefit guarantee that is included with many contracts, see Table 1). More specifically, in late 2019 and early 2020, under relatively normal financial market conditions, RILA policies were priced fairly homogeneously to yield the carriers a gross surplus of 1.6% to 3.3%.[23] As expected, contracts that do not include a GMDB rider tend to be on the lower end of the surplus range.

Figure 2 plots the gross surplus Π_t^{gross} of all 29 sample contracts, categorized by carrier. The *full* markers represent the contracts pre-pandemic and the figure shows that under these relatively normal economic conditions the surplus was quite homogeneous, especially within each provider. Moreover, Table 2 shows that the carriers' average profitability is roughly the

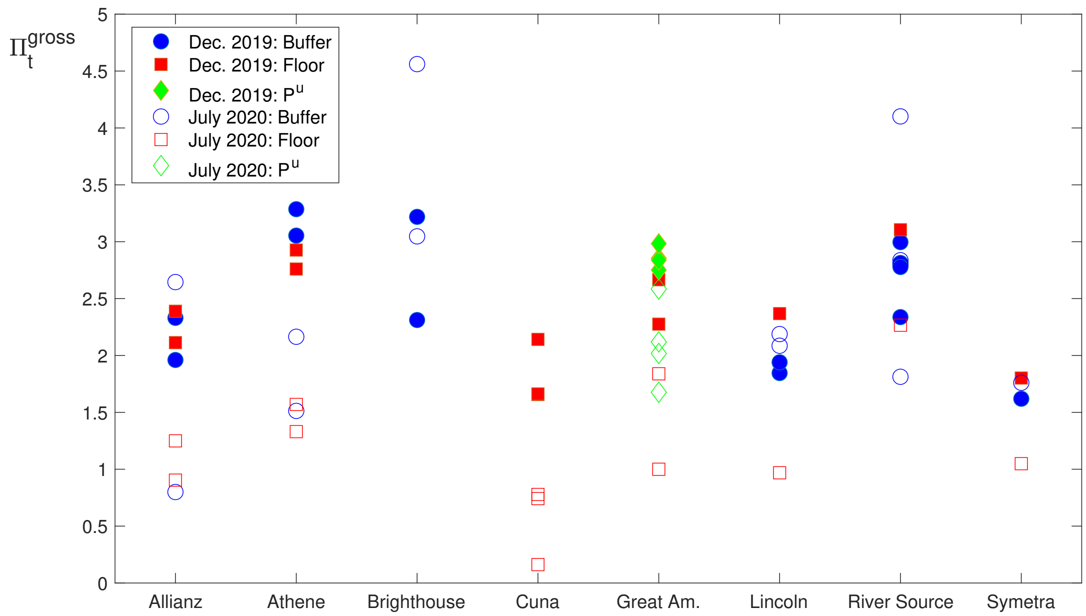
[20] A large part of the carriers' expenses are the policy acquisition costs which the carrier pays to the agent or broker that brings in the policy, and the magnitude and structure of this expense may vary based on the sales channel and the carrier's arrangements with agents and brokers. The carrier then expects to recover these up-front expenses from the investor indirectly and in small installments year by year. Therefore, the insurer's net gains from the contract (after expenses) depend critically on how long the investors remain invested in their contracts. This can vary greatly by provider as well.

[21] Source: *S&P Global – Market Intelligence*. The rate is defined as the "annualized investment return based on average invested assets".

[22] The annualized return on the 1-month T-bill was 2.14% in 2019, as reported by Kenneth French (Source: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research`, accessed on February 17, 2021).

[23] Nonetheless, the current pricing structure may leave carriers exposed to credit risk, unless they choose to pass this risk on to their policyholders retroactively by lowering the cap rates in the years following sub-par returns on their fixed-income investments.

Figure 2: Gross Surplus by Carrier



Note: The figure plots the annualized gross surplus Π_t^{gross} of the 29 sample contracts, sorted by carrier. The filled markers reflect the surplus under "normal" times (December 31, 2019), while the empty markers represent the surplus of the same contracts during a high-volatility low-interest rate period (July 1, 2020). The shape of the marker depends on the contract's downside protection method, using a circle for a buffer, a square for a floor and a diamond for an upside probability. The 29 contracts are described in Table 1 and Π_t^{gross} is defined in Equation (8).

same for buffer contracts as for contracts with a floor (250 versus 232 basis points). This homogeneity is not particularly surprising given that (i) providers can eliminate virtually all of the embedded risk through the financial markets, and (ii) in contrast to TVAs, the general structure of the RILA contracts makes them relatively easy to compare.

Table 2 also shows that during the Covid-19 pandemic almost all carriers priced their RILA products (even) more favorably for the investors, with a reduced gross surplus of 1.9% on average. As market volatility increased drastically, in-the-money put and call options became much more expensive. As discussed above, these changes had a positive effect on RILAs with a *buffer*, allowing insurers to offer larger cap rates in July 2020 (see Table 1) in order to maintain approximately the same level of profitability (on average 250 basis points in December 2019 versus 249 basis points in July 2020, see Table 2). In contrast, for RILAs with a *floor*, the increase in put option prices was detrimental and required insurers to *reduce* the cap rates in July 2020. However, the profitability estimates in Table 2 suggest that insurers chose to absorb some of the financial consequences of the current conditions as they did not decrease the rates to the full extent and saw their estimated surplus decline by over a percentage point on average. Figure 2 visualizes these insights. The *empty* markers represent the profitability of the same 29 contracts *during* the pandemic. In particular, we can see that buffer contracts (blue circles) are similarly profitable before and during the pandemic, while carriers reduced their surplus on floor contracts (red squares) fairly consistently by around one percentage point. This leads to an increased level of heterogeneity, even within providers.

4.3 Discussion

Notably, the above results are independent of modeling and parameter assumptions—with the exception of each carrier's expected risk premium, and even that is empirically estimated from recent data—and thus offer unique insights into insurers' pricing when virtually all risks can be eliminated at little cost, and into how they respond in times of crisis.

Here, an interesting question is what motivated insurers to adjust their RILA guarantees in this particular way to the increased market volatility levels in mid-2020. Koijen and Yogo (2015) observe that during the 2008 financial crisis life insurers sold long-term contracts far below their actuarially fair value. They attribute this to a combination of incentives provided by accounting regulations and the need to rebuild the firms' balance sheets after the crisis.[24] I do not believe that this was the case for RILA carriers in mid-2020, as that should have affected *both* floor *and* buffer contracts. However, only the floor contracts saw a price reduction (in the form of a negative cost C_t).

Instead, this situation feels more closely related to the price stickiness in insurance that has been observed e.g. by D'Arcy and Doherty (1986) in a multi-period contracting setting and by Charupat et al. (2016) who document a relatively slow and asymmetric adjustment of life annuity prices in response to changing interest rates. Both of these scenarios apply in some way to RILA contracts (with changes in equity volatility in addition to interest rates). The increase in equity volatility (and lower interest rates) made floor contracts more expensive and made buffer contracts less expensive (ceteris paribus). For the buffer contracts, insurers generally increased the cap rates (i.e., lowered the prices) immediately to retain the same level of profitability. In contrast, for floor contracts insurers lowered the caps (i.e., increased the prices) though not as much as needed to maintain the pre-pandemic level of profitability. This is very similar to the findings of Charupat et al. (2016) that insurers are quick to lower prices when a product's actuarial value declines but are more hesitant

[24] However, in a recent study, Bauer et al. (2021) reexamine this situation and come to a very different conclusion.

to increase prices under contrary circumstances. In the present case of the RILA contracts, insurers likely expected the financial markets to return to "normal" in the following year and did not want to show excessive negative fluctuations in their cap rates over time in order to ensure current and future investors of the financial stability of this product.

Going beyond the above pricing model, RILAs offer providers other potential benefits that could help explain why many are willing to sell the product with such generous guarantees. First, it may not be necessary or optimal to fully hedge the RILA guarantees with financial options because they can provide a partial natural hedge to TVAs: if the financial market performs well during a given period, insurers tend to profit from their TVA offerings—because they collect more fees while the value of the embedded guarantees declines—but they have to credit a return to their RILA accounts. Conversely, if the market generates a negative return, then the RILA accounts (i.e., the insurer's liabilities) decline in value, while the insurer's assets (which are invested in fixed-income securities) likely will not. Under this financial scenario, the profit from the RILA contracts can help offset the increase in the insurer's TVA liabilities. Therefore, the presence of RILA products can reduce the hedging costs and the capital requirements for the insurers' TVA offerings—both of which are often substantial due to the long-term nature of the TVA guarantees. This might also explain why insurers are offering RILAs with longer-term guarantees ($3-6$ years), even though they cannot easily hedge their resulting liabilities since the financial markets do not trade options with those maturities. Second, even if they fully hedge their RILA liabilities, insurers may benefit from pooling them with their in-force TVA policies due to the *diversification* between the fixed-income risk of the former and the equity risk of the latter. This could potentially reduce the firm's reserve and/or capital requirements. The interaction between RILAs and TVA contracts in a life insurer's portfolio presents an interesting avenue for future research.

Third, and related to that, providers may try to make RILAs particularly attractive in order to entice current TVA policyholders to exchange their contract for a RILA product or to attract potential investors to RILAs instead of TVAs. Since the upcoming changes to GAAP accounting principles will likely have a major negative impact on insurers who have a lot of long-term risk exposure from TVAs on their books, the short-term nature of RILAs may make them (at least temporarily) preferable for insurers, even if they may be less profitable on their own. In addition, increasing the assets under management from RILA contracts would produce a better balance with their TVA assets so that they can take greater advantage of the natural hedge (or the diversification benefits) between the two product types, as described above.

And finally, it is also conceivable that insurers "lure in" new investors with generous offers and then reduce the caps over time. However, there is no evidence of that thus far as current cap rates for *renewal* policies tend to be even higher than the caps for *new* policies. Whatever the insurers' reasoning behind their current offerings, RILAs appear to be attractively priced for investors who are willing to accept some short-term financial risks.

5 Numerical Analysis: Long-Term Payout Profiles

This section provides a numerical illustration of the distribution of payouts to the RILA investor over the long term, for different protection and crediting mechanisms and in comparison to other equity-linked annuity products. To that effect I conduct a Monte Carlo simulation (with one million paths) for a stock index over 18 years, and determine the accumulated account value under each path of an initial nominal investment of $100 that is allocated to this index and protected by different financial guarantees. These guarantees reflect common products in the U.S. market. Specifically, I implement the following contracts for comparison:

[1] A RILA with one-year terms, a 10% buffer and a 16.3% cap, with no annual fee.

[2] A RILA with one-year terms, a 10% floor and a 20.9% cap, with no annual fee.

[3] A RILA with six-year terms, a 15% buffer and a 350% cap, with no annual fee.

[4] A RILA/FIA with one-year terms, full downside protection (i.e., a 100% buffer or a 0% floor), and a 4.9% cap, with no annual fee.

[5] A TVA with a return-of-premium maturity guarantee (RoP GMMB) and a combined annual fee of 280 bps charged at the beginning of each year (120 bps base fee + 30 bps mutual fund fee + 130 bps guarantee fee).

[6] A TVA with a six-year return-of-premium guaranteed minimum accumulation benefit (GMAB). After the six-year period, the policy pays the larger of the current account value and the initial investment. The policy is renewed twice, each time with the last payout as the new starting account value. However, the policyholder may only invest up to 60% of the funds in equity, with the remaining 40% placed in a risk-free asset. The annual fee rate is 200 bps for the guarantee and thus 350 bps total.

Since I'm interested in comparing the payouts at maturity, I assume no mortality and no withdrawals or surrenders along the way. Therefore, all six contracts produce a payout at time 18 along each simulated path. For the simulation, I assume a Black-Scholes model[25] with a constant risk-free rate of return of 3%, a constant annual volatility of 20%, and a constant expected rate of return of the average stock in the index of 9%. Moreover, the stocks pay dividends at an average rate of 2% per year. The index underlying the four RILA/FIA products is a price index and does not reflect the dividend payments; therefore, the index grows at an expected rate of 7%. In contrast, the two TVA contracts are invested in a mutual fund that contains the stocks themselves, and that has all dividend payments from those stocks immediately reinvested. Thus, the funds underlying the TVA accounts earn the full 9% on average.

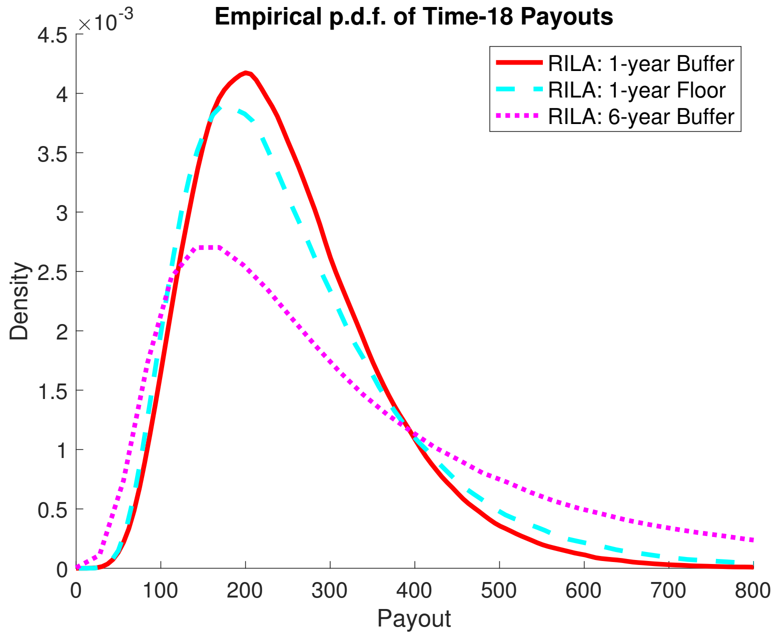
In order to achieve a meaningful comparison of the payouts across the six products, I made the following assumptions: For the TVAs, I set the rates of the guarantee fee such that the insurer's risk-neutral present value of the guarantee payments equals the guarantee fee income. For the RILA contracts, I chose the cap rates such that the risk-neutral present value of the investor's terminal payout from the product equals approximately 85% of the initial investment. This leaves the insurer with gross earnings of 15% of the investment (i.e., around 1% per contract year), based on a market-consistent valuation approach.[26] Notably, these equity-linked annuity products can still be attractive to the investor due to the embedded tax benefits (see e.g. Brown and Poterba, 2006; Moenig and Bauer, 2016). A more formal description of the Monte Carlo simulation and the pricing assumptions can be found in Appendix B.

The results of the simulation analysis are shown in Figure 3 and Table 3. They concur with general intuition but also provide interesting practical insights. First, the correlation matrix at the bottom part of Table 3 shows that the terminal values of the six contracts tend to be highly correlated, often near or above 0.90. This makes sense because all payouts are positively correlated with the performance of the same underlying index. It is also expected that the correlations are weaker for the FIA contract [4] due to its severe limitations of the upside and downside risk.

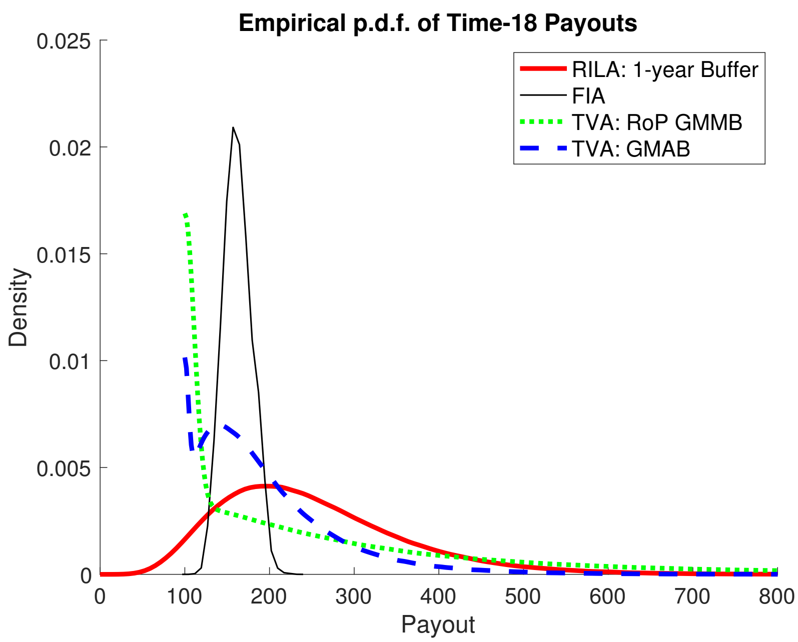
[25]While the Black-Scholes model is an oversimplified representation of a stock index' performance, I apply it here for two reasons: First, as mentioned e.g. by Célérier and Vallée (2017) and Egan (2019), it provides a conventional benchmark that is not subject to unstable parameter choices. This is particularly relevant given the long-term nature of the investments considered in this analysis. And second, in a Black-Scholes framework, the RILA guarantees have the same value every term. This allows the cap rate to remain constant for the purpose of this illustration.

[26]For comparison, the risk-neutral present value of payouts from the two TVA products is 78% and 77% of the premium, respectively. However, TVAs are known for their high fee rates, and the results of the previous section imply that RILA products, at least in their current form, are less costly. Nonetheless, RILA providers also face considerable acquisition expenses and other costs, which must be reflected in the pricing of the product, at least in the long term. Therefore, I believe that the assumption of an 85% (risk-neutral) value to investment ratio is reasonable.

Figure 3: Empirical p.d.f. of Time-18 Payouts



(a) Different RILA Contracts



(b) RILA vs. FIA vs. TVA

Notes: The figure shows the empirical probability distribution function (p.d.f.) of the maturity payouts from each of the six equity-linked annuity contracts described at the beginning of this section. The p.d.f. is computed from the outcomes of a Monte Carlo simulation with 1 million sample paths, using kernel smoothing. Panel (a) compares the three RILA contracts with each other, and panel (b) compares a representative RILA product with other equity-linked annuities.

Table 3: Payout Statistics for Simulated Contracts

	[1]	[2]	[3]	[4]	[5]	[6]
	RILA 1-year term w/ buffer	RILA 1-year term w/ floor	RILA 6-year term w/ buffer	FIA 1-year term	TVA w/ RoP GMMB	TVA w/ GMAB
Effective annual return:						
Mean	0.0471	0.0479	0.0598	0.0268	0.0485	0.0341
St.dev.	0.0257	0.0289	0.0437	0.0056	0.0422	0.0217
Time-18 payouts:						
Mean	250.39	260.71	371.93	161.58	309.42	196.32
St.Dev.	110.04	136.15	337.81	15.77	306.21	85.42
P-tile 1	75.30	73.10	57.40	127.60	100.00	100.00
P-tile 5	107.00	101.80	88.00	136.80	100.00	104.40
P-tile 10	127.90	121.60	110.50	141.50	100.00	114.30
P-tile 25	170.60	164.70	166.60	150.60	119.30	137.30
Median	231.60	230.70	271.20	161.30	211.20	174.80
P-tile 75	309.70	323.00	454.90	171.70	373.90	230.50
P-tile 90	396.70	436.20	737.10	182.40	625.20	302.40
P-tile 95	456.60	519.90	986.80	188.10	852.20	358.60
P-tile 99	588.30	720.70	1,700.60	201.00	1,520.10	499.50
Prob. $<$ 100	0.0381	0.0468	0.0754	0	0	0
Prob. $>$ 200	0.6285	0.612	0.6611	0.0109	0.5256	0.3686
Prob. $>$ 500	0.0302	0.0593	0.2139	0	0.1547	0.0101
Correlation matrix of payouts:						
[1]	1					
[2]	0.9145	1				
[3]	0.8429	0.8701	1			
[4]	0.8355	0.8888	0.6967	1		
[5]	0.8218	0.8509	0.9857	0.6673	1	
[6]	0.8365	0.8677	0.9644	0.7174	0.9446	1

Notes: The table displays statistics for the annualized return and the terminal payout of the 18-year investment under the six equity-linked annuity contracts, as described at the beginning of this section. Results are based on a Monte Carlo simulation with 1 million sample paths, with a nominal initial investment of 100.

Second, Figure 3(a) contrasts the three RILA contracts and shows that the 1-year buffer and the 1-year floor produce a relatively similar distribution of terminal payouts, with the floor protection being slightly riskier. In contrast, the RILA with 6-year terms is considerably riskier for the policyholder as the infrequent buffering leaves more room for large negative returns, while the higher cap gives the investor a greater upside potential.

Third, Figure 3(b) confirms the expectation that the FIA contract is the least risky of the six investment vehicles. In line with that, Table 3 further shows that this contract also earns the lowest average rate of return. In fact, the FIA gives the investor only a 1% chance to double its money over the 18-year investment. In contrast, this probability exceeds 60% for each of the RILA contracts, which is higher than for the two TVAs.

And fourth, the two TVA products have somewhat distinct payout distributions. In fact, the return-of-premium GMMB rider is considerably riskier than the GMAB policy, since the reduced equity exposure on the latter—which was necessary to keep the guarantee fee at a reasonable rate—limits the GMAB's upside potential. In fact, the distribution of gains for the TVA+GMAB contract appears to be quite similar to that of the RILA with a 1-year buffer (see Figure 3(b) and the percentile statistics of Table 3). Moreover, the 6-year RILA contract possesses roughly the same upside potential—and even a little bit more—than the TVA+GMMB contract (see Table 3). These two are by far the riskiest of the six investments, with the TVA+GMMB having the additional benefit of full downside protection. In contrast, all three RILA products do expose the investor to the risk of an overall financial loss. However, I find that while a negative credited return is easily possible in any given term, it is relatively unlikely that the investor actually loses money on this investment over the full 18-year term, with estimated probabilities between 3.8% and 7.5%.

In conclusion, this simulation exercise reveals that over the long term the payout profiles of RILA contracts are comparable to TVA policies with long-term guarantees. While RILAs allow for temporary losses on the investment, this risk is largely diversified away through time, leaving only a small chance (around 5%) of a long-term financial loss. On the other hand, in terms of the potential to take advantage of strong equity markets, RILAs can easily compete with standard TVA products. Thus, for individuals who have a long-term investment horizon and are looking for equity exposure with some downside protection and preferential tax treatments, RILAs may well be the product of choice.

6 Conclusion

This study describes and analyzes RILA products, a recent type of equity-linked annuities that has been gaining considerable popularity in the U.S. market. I show that over the long term, RILAs offer investors an upside potential that is comparable to TVA policies, while their downside protection limits the likelihood of a financial loss. Crucially, however, RILAs are much cheaper and easier for insurers to manage as the firms' liabilities can be decomposed into short-term European options on popular market indexes. That is, while the long-term guarantees embedded in TVA contracts have to be hedged "manually" and still expose providers to considerable equity risk, RILAs with short guarantee terms of one or two years can be hedged near-perfectly by purchasing standard European options in the financial markets. As a result, RILAs can provide carriers with a highly predictable and stable profit, permitting the product to be offered at a low cost. These features make RILAs a suitable product to reinvigorate the equity-linked annuity market.

In fact, RILAs may play a particular role in QRPs since they offer investors looking to save for retirement equity exposure with downside protection, and they do so at little cost and in a relatively transparent fashion. These features make RILAs ideally suited to be default options (QDIAs) in employer-sponsored DC plans. Moreover, RILAs are flexible and diverse enough to be tailored to a client's retirement planning, similar to lifecycle funds. Comparing these two strategies (in terms of cost, utility to investors, etc.) should be an interesting research project that can be of great value to the public.

One of the more intriguing results of this study is that during the 2020 pandemic carriers were willing to absorb a big part of the increased hedging cost for RILA contracts with a floor protection. While floor and buffer contracts produced very similar levels of profitability pre-pandemic, the floor contracts yielded over 100 basis points less in profitability in mid-2020. It would be interesting to assess the carriers' decision making in this context.

Another worthwhile topic for future research is the integration of RILAs and TVA into a "variable annuity" portfolio. The TVA market contains approximately $2 trillion in net assets, TVAs comprise the largest category of liabilities for many U.S. life insurers (Koijen and Yogo, 2018) and they entail high costs for hedging, reserving and capital requirements. As I discussed above, RILAs can act as a (partial) natural hedge to the insurers' liabilities from TVA guarantees, in addition to providing some diversification benefit. Gaining a better understanding of the interaction between the two product types, how effective the natural hedge can be, and what an ideal "variable annuity" hedging strategy might look like would be an important next step.

And finally, most RILA products include a return-of-premium death benefit guarantee during the payout phase, at no cost. Some providers have also started to offer GLWB riders for an additional fee (AM Best, 2019). How these TVA-style guarantees interact with the different account crediting methods and hedging strategies of RILAs presents another interesting opportunity for future research.

Appendices

A Proofs

Combining Equation (2) and Equation (4) gives us:

$$L_{t+\tau} = -A_t\,(b_t - 1) + A_t\,\epsilon\,\tau\,b_t - A_t\,\varphi\,\tau + \frac{A_t\,(1-\varphi\,\tau)}{S_t}\cdot\left(S_t\,R_t^A\right) - H_{t+\tau} + H_t\,b_t\,. \tag{9}$$

Also, for notational convenience, I abbreviate $(x)_+ = \max\{x, 0\}$.

A.1 Proof of Equation (4B)

If the RILA contains a buffer $B > 0$, then by Equation (3):

$$\begin{aligned}
S_t\,R_t^A &= S_t\cdot\min\{R_t^S + B, 0\}\cdot\mathbb{1}_{\{R_t^S<0\}} && +P^u\,S_t\cdot\min\{R_t^S, C\}\cdot\mathbb{1}_{\{R_t^S\geq 0\}}\\
&= -S_t\cdot\left(-B - R_t^S\right)_+\cdot\mathbb{1}_{\{R_t^S<0\}} && +P^u\,S_t\cdot\left(R_t^S + C - \max\{R_t^S, C\}\right)\cdot\mathbb{1}_{\{R_t^S\geq 0\}}\\
&= -S_t\cdot\left(-B - R_t^S\right)_+ && +P^u\,S_t\,R_t^S\cdot\mathbb{1}_{\{R_t^S\geq 0\}} - P^u\,S_t\cdot\left(R_t^S - C\right)_+\cdot\mathbb{1}_{\{R_t^S\geq 0\}}\\
&= -S_t\cdot\left(-B - R_t^S\right)_+ && +P^u\,S_t\,\left(R_t^S\right)_+ - P^u\,S_t\cdot\left(R_t^S - C\right)_+\\
&= -\left(S_t(1-B) - S_t(1+R_t^S)\right)_+ && +P^u\cdot\left(S_t(1+R_t^S) - S_t\right)_+ - P^u\cdot\left(S_t(1+R_t^S) - S_t(1+C)\right)_+\\
&= -\left(S_t(1-B) - S_{t+\tau}\right)_+ && +P^u\cdot\left(S_{t+\tau} - S_t\right)_+ - P^u\cdot\left(S_{t+\tau} - S_t(1+C)\right)_+\\
&= -\mathrm{Put}_{t+\tau}(S_t(1-B)) && +P^u\cdot\mathrm{Call}_{t+\tau}(S_t) - P^u\cdot\mathrm{Call}_{t+\tau}(S_t(1+C))\,.
\end{aligned}$$

Plugging this identity into Equation (9) produces Equation (4B), as desired. □

A.2 Proof of Equations (4F) and (4F')

Similarly, if the RILA offers downside protection through a floor $F > 0$, by Equation (3):

$$\begin{aligned}
S_t\,R_t^A &= S_t\cdot\max\{R_t^S, -F\}\cdot\mathbb{1}_{\{R_t^S<0\}} && +P^u\,S_t\cdot\min\{R_t^S, C\}\cdot\mathbb{1}_{\{R_t^S\geq 0\}}\\
&= S_t\cdot\left(\max\{0, -F - R_t^S\} + R_t^S\right)\cdot\mathbb{1}_{\{R_t^S<0\}} && +P^u\,S_t\cdot(R_t^S)_+ - P^u\,S_t\cdot(R_t^S - C)_+\\
&= S_t\cdot\left(-F - R_t^S\right)_+ - S_t\cdot\left(-R_t^S\right)_+ && +P^u\,S_t\cdot\left(R_t^S\right)_+ - P^u\,S_t\cdot\left(R_t^S - C\right)_+\,.\\
&= \left(S_t(1-F) - S_t(1+R_t^S)\right)_+ && +P^u\cdot\left(S_t(1+R_t^S) - S_t\right)_+\\
&\quad - \left(S_t - S_t(1+R_t^S)\right)_+ && -P^u\cdot\left(S_t(1+R_t^S) - S_t(1+C)\right)_+\\
&= (S_t(1-F) - S_{t+\tau})_+ - (S_t - S_{t+\tau})_+ && +P^u\cdot(S_{t+\tau} - S_t)_+ - P^u\cdot(S_{t+\tau} - S_t(1+C))_+\\
&= \mathrm{Put}_{t+\tau}(S_t(1-F)) - \mathrm{Put}_{t+\tau}(S_t) && +P^u\cdot\mathrm{Call}_{t+\tau}(S_t) - P^u\cdot\mathrm{Call}_{t+\tau}(S_t(1+C))\,.
\end{aligned}$$

Plugging this identity into Equation (9) produces Equation (4F), as desired. □

And for the special case of $P^u = 1$, we can simplify the above calculations, beginning in line 5:

$$\begin{aligned} S_t\, R_t^A = \; & \underbrace{(S_t(1-F) - S_{t+\tau})_+}_{=S_t(1-F)-S_{t+\tau}+(S_{t+\tau}-S_t(1-F))_+} \underbrace{- (S_t - S_{t+\tau})_+ + (S_{t+\tau} - S_t)_+}_{=S_{t+\tau}-S_t} - (S_{t+\tau} - S_t(1+C))_+ \\ = \; & S_t(1-F) - S_{t+\tau} + (S_{t+\tau} - S_t(1-F))_+ + S_{t+\tau} - S_t - (S_{t+\tau} - S_t(1+C))_+ \\ = \; & -S_t\, F + \text{Call}_{t+\tau}(S_t(1-F)) - \text{Call}_{t+\tau}(S_t\,(1+C))\,, \end{aligned}$$

which proves Equation (4F'). □

A.3 Proof of Equation (4P)

Lastly, if the protection is the result of a downside participation rate $P^d < 1$, Equation (3) implies that:

$$\begin{aligned} S_t\, R_t^A = \; & P^d \underbrace{S_t \cdot R_t^S \cdot \mathbb{1}_{\{R_t^S < 0\}}}_{=-S_t \cdot (-R_t^S)_+ = -(S_t - S_{t+\tau})_+} + P^u\, S_t \cdot \min\{R_t^S, C\} \cdot \mathbb{1}_{\{R_t^S \geq 0\}} \\ = \; & -P^d \cdot \text{Put}_{t+\tau}(S_t) + P^u \cdot \text{Call}_{t+\tau}(S_t) - P^u \cdot \text{Call}_{t+\tau}(S_t(1+C))\,. \end{aligned}$$

Plugging this identity into Equation (9) produces Equation (4P), as desired. □

B RILA Payout Profiles: Implementational Details

In this section I provide some additional details and a more formal setting for the Monte Carlo simulation used in the final "Numerical Analysis" section.

Let S_t denote the time-t value of the stock index. In the Black-Scholes framework, the change in the index value is then given by the stochastic differential equation

$$dS_t = (\mu - \delta)\, S_t\, dt + \sigma\, S_t\, dW_t\,,$$

where μ denotes the expected rate of return of the average stock in the index, δ is the annual dividend yield, σ is the annual volatility of the index' log return, and W_t is a standard Brownian motion.

The net return of the index is then given as R_t^S by Equation (1). The resulting return credited to each RILA contract over the crediting term is specified by Equation (3), with the RILA account value A_t updated as in Equation (2). To price the RILA contracts [1], [2], [3] and [4], I determined the cap rate C for each contract such that the discounted expected payout to the policyholder equals 85% of the initial investment (as discussed in the main text). That is:

$$e^{-r \cdot 18} \cdot \mathbb{E}^{\mathbb{Q}}\left[A_{18}\right] = 0.85 \cdot A_0 \,,$$

where r denotes the annual risk-free rate of return. To estimate C numerically, I conduct a Monte Carlo simulation with one million sample paths for S_t (and thus A_t) under the risk-neutral measure $\mathbb{Q}$, that is with $\mu = r$. The expected value in the above equation is then approximated by the average of the accumulated account values in the sample.

For the TVA policies, the account value A_t is updated year-by-year according to

$$A_{t+1} = A_t \cdot (1 - \varphi^{\mathrm{TVA}}) \cdot \exp\left(\alpha \cdot \left(\ln\left(\frac{S_{t+1}}{S_t}\right) + \delta\right) + (1-\alpha) \cdot r\right) \,, \tag{10}$$

where φ^{TVA} denotes the annual fee rate on the TVA policy and α represents the contract's equity exposure. The δ-term reflects the immediate reinvestment of the stocks' dividend payments. The insurer collects guarantee fees of amount $A_t \cdot \varphi^{\mathrm{guar}}$ at the beginning of each policy year, that is at times 0, 1, …, 17.[27] For the purpose of this study I determine the guarantee fee rate φ^{guar} such that the guarantee fees have the same expected present value (under the risk-neutral measure $\mathbb{Q}$) as the insurer's payout from the guarantee. That is, in the case of contract [5]:

$$\mathbb{E}^{\mathbb{Q}}\left[\sum_{t=0}^{17} e^{-r\,t} \cdot A_t \cdot \varphi^{\mathrm{guar}}\right] = e^{-18 \cdot r} \cdot \mathbb{E}^{\mathbb{Q}}\left[\max\{A_0 - A_{18}, 0\}\right] \,.$$

For contract [6], the TVA account value will be stepped up to the guaranteed amount at times 6 and 12, that is:

$$A_6 = \max\{A_{6^-}, A_0\} \quad \text{and} \quad A_{12} = \max\{A_{12^-}, A_6, A_0\} \,,$$

with A_{t^-} denoting the TVA account value at time t just prior to the potential step-up. The pricing identity for contract [6] is then given by:

$$\mathbb{E}^{\mathbb{Q}}\left[\sum_{t=0}^{17} e^{-r\,t} \cdot A_t \cdot \varphi^{\mathrm{guar}}\right] = \sum_{t \in \{6,12,18\}} e^{-r\,t} \cdot \mathbb{E}^{\mathbb{Q}}\left[\max\{A_0 - A_t, 0\}\right] \,.$$

[27]Note that φ^{TVA} is the product's baseline fee rate, while φ^{guar} is the fee rate for the (usually optional) add-on rider. The former helps insurers recover their general expenses (sales commissions, etc.) and only affects the evolution of the account value; see Equation (10). In contrast, φ^{guar} should be set so as to provide the insurer with enough income (in expectation) to cover the payout from the elected guarantees. This valuation identity is reflected in the following equations.

The expected values are again approximated with a standard Monte Carlo simulation.

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