Precautionary motives with multiple instruments*

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Abstract

We use a unified approach to show how precautionary saving, self-protection and self-insurance are jointly determined by risk preferences and the preference over the timing of uncertainty resolution. We examine higher-order risk effects and analyze risk averters and risk lovers. When decision-makers use several instruments simultaneously, substitutive interaction effects arise. We quantify precautionary and substitution effects and discuss the role of instrument interaction for the inference of preference parameters from precautionary choices. Instruments can differ substantially in the size of the precautionary motive and in the susceptibility to substitution effects. This affects their suitability for the identification of precautionary preferences.

Keywords: Recursive preferences · prudence · precautionary behavior · interaction effects · comparative statics

JEL-Classification: D11 · D80 · D81 · G22

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1 Introduction

The idea that uncertainty about future income raises saving goes back to Keynes and Hicks and was first analyzed theoretically in the late 1960s by Leland (1968), Sandmo (1970) and Drèze and Modigliani (1972). Decision-makers who behave in this way are called *prudent*. Ever since Kimball's (1990) seminal paper, we know that prudence has a simple and intuitive characterization in the additively separable expected utility model: a convex marginal utility of future consumption, $u''' \geq 0$. The notion of prudence and precautionary motives more generally play important roles in microeconomics, macroeconomics and asset pricing.

In this paper, we extend the analysis of precautionary behavior in various ways. In particular, we follow Kimball and Weil (2009) and use recursive utility proposed by Kreps and Porteus (1978) and Selden (1978) to disentangle risk and time. We study various tools that decision-makers can use to react to uncertainty, which we call *instruments*, and consider saving but also self-protection and self-insurance (see Ehrlich and Becker, 1972). We derive a unifying result on how risk preferences and the preference over the timing of uncertainty resolution jointly determine prudence. We discuss higher-order risk effects (see Eeckhoudt and Schlesinger, 2006, 2008; Noussair et al., 2014) because income risk often involves changes beyond the second order. We also examine the behavior of both risk averters and risk lovers (see Crainich et al., 2013; Deck and Schlesinger, 2014) because both types of decision-makers occur in the data. For choices in one instrument, our analysis thus broadens the scope of the literature on precaution considerably.

Empirically it appears more plausible that decision-makers use more than one instrument to respond to uncertainty. In this case, interaction effects arise and general predictions about precautionary behavior are difficult to derive. We supplement our theoretical results with a detailed numerical analysis. Instruments differ in the intensity of precautionary motives and their susceptibility to substitution effects. Precautionary self-protection can be *decreasing* in income risk precisely because decision-makers also engage in saving and self-insurance. This shows clearly that the link between preferences and precautionary behavior may depend critically on the portfolio of instruments used by the decision-maker. Interaction effects can also distort the inference of preference parameters from precautionary behavior, and the size of this distortion can be large. Low levels of precautionary self-protection or precautionary self-insurance may thus suggest *negative* values for relative prudence because saving absorbs most of the precautionary response. From a practical perspective, instruments differ in how well they are suited to infer preferences from precautionary motives.

Our analysis is motivated by recent interest in instruments other than saving that are subject to precautionary income risk effects. We draw on Ehrlich and Becker's (1972) distinction between self-protection, a costly activity to reduce the probability of loss, and self-insurance, a costly activity to reduce the severity of loss.[1] Eeckhoudt et al. (2012), Courbage and Rey

[1] Courbage et al. (2013) review the literature on self-protection and self-insurance and give many examples.

(2012) and Wang and Li (2015) analyze precautionary self-protection effort as a characterizing trait for prudence, much like precautionary saving, suggesting self-protection as a viable alternative to identify precautionary preferences. They use the additively separable expected utility model, which collapses relative risk aversion and the resistance to intertemporal substitution of consumption.[2] A few studies have also looked at precautionary (self-)insurance in atemporal expected utility settings (see Eeckhoudt and Kimball, 1992; Fei and Schlesinger, 2008) or in the additively separable expected utility model (Wang et al., 2015; Wong, 2016). As argued by Kimball and Weil (2009), this model does not allow us to ask questions *"that are fundamental to the understanding of consumption in the face of labor income risk,"* which is why we employ recursive utility to disentangle risk preferences from time preferences. We can then distinguish between a preference for late versus early resolution of uncertainty, which matters descriptively (e.g., von Gaudecker et al., 2011) and turns out to affect our results. Labor income risk reduces (increases) certainty-equivalent consumption for risk averters (risk lovers), which stimulates precautionary behavior for decision-makers who prefer a late (early) resolution of uncertainty. If risk preferences satisfy mutual aggravation of risk increases (see Eeckhoudt et al., 2009; Ebert et al., 2018), the marginal value of risk reduction increases and prudent behavior arises. Our results organize, unify and extend existing results about precautionary saving, self-protection and self-insurance under recursive utility.

This commonality raises the question about precautionary behavior when decision-makers use more than one instruments at a time to respond to income risk. In this regard, our paper is related to the literature on precautionary saving with endogenous labor supply. Based on a calibrated life-cycle model, Low (2005) finds that labor-supply flexibility leads to more borrowing among young households and more precautionary saving among the middle-aged. Flodén (2006) finds greater precautionary saving with endogenous labor supply in a two-period model with a utility function that satisfies balanced growth. Nocetti and Smith (2011) extend Flodén's results to recursive utility and income risks of any size. Under plausible conditions, the complementarity between saving and labor supply outweighs the hedging effect of labor-supply flexibility.[3] In the case of saving, self-protection and self-insurance considered here, substitution effects arise and diminish the precautionary use of each instrument. In our

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Our model covers any type of safety investment that households make to mitigate property and liability risks or the financial impact of adverse health outcomes. Insurance demand is a special case of self-insurance and thus yields many additional examples. Against this background, self-protection and self-insurance activities are ubiquitous in household financial decision-making.

[2] Wang et al. (2019) is the only paper we are aware of that analyzes precautionary self-protection with Kreps-Porteus/Selden preferences. They do not consider other instruments, interaction effects, or the inference of preference parameters from precautionary motives.

[3] A rich literature that is too extensive to be summarized here, has studied precautionary responses to income risk under incomplete markets. Zeldes (1989) derives closed-form solutions for optimal consumption with stochastic labor income, Deaton (1991) analyzes the effect of liquidity constraints on precautionary saving, and Gourinchas and Parker (2002) decompose saving into its precautionary and life-cycle components based on an estimated structural model. Low et al. (2010) distinguish between different types of labor income risk and Heathcote et al. (2014) use labor supply decisions to quantify risk sharing of idiosyncratic shocks.

numerical analysis, these substitution effects can be large enough to outweigh precautionary effects and thus lead to precautionary disinvestment, even for plausible choices of preference parameters and risk levels. So while the literature on precautionary saving with endogenous labor supply tends to find complementarity, we find the opposite.

Interaction effects also arise for joint saving and insurance decisions. In a two-period model with non-separable utility, saving and insurance are pure substitutes in the Hicks sense when the utility function satisfies decreasing temporal risk aversion (Dionne and Eeckhoudt, 1984). Based on continuous time life-cycle models, Briys (1986, 1988) shows that consumption and insurance decisions are only separable under restrictive assumptions on the utility function, and Gollier (1994, 2003) finds that precautionary wealth accumulation may dominate insurance in the long-run.[4] A number of empirical studies confirm the relationship between saving and insurance, and document lower levels of precautionary saving when individuals are more comprehensively insured (see Gruber and Yelowitz, 1999; Engen and Gruber, 2001; Chou et al., 2003).[5] We provide a preference-based foundation of these interaction effects and show that they arise for *any* combination of instruments that trade off current consumption against (expected) future consumption. The case of saving and insurance is a good example but the underlying mechanism is substantially more general.

Our paper also contributes to the literature on the prevalence and strength of precautionary motives in the field and the underlying preferences. This literature finds a variety of results and faces some methodological challenges. Based on the Consumer Expenditure Survey Data, Dynan's (1993) largest point estimate for relative prudence is 0.312. She concludes that *"[w]e cannot reject the hypothesis that the coefficient of relative prudence is zero."*[6] Merrigan and Normandin (1996) find relative prudence ranging from 1.78 to 2.33 based on longitudinal expenditure data from the UK. Eisenhauer (2000) states a range from 1.51 to 5.15 using survey data on life insurance, and Eisenhauer and Ventura (2003) report values from 7.32 to 8.65 based on hypothetical choices. In a survey of the empirical literature, Lugilde et al. (2019) point out the lack of consensus regarding the intensity of the precautionary saving motive. The presence of multiple instruments may help explain this issue. When decision-makers respond to income risk broadly by adjusting several instruments, substitution effects diminish the amount of precautionary saving, self-protection and self-insurance. Especially in the field,

[4] Somerville (2004) modifies Briys' approach to study the dynamic effects of the loss probability on precautionary saving. His results corroborate the importance of interaction effects between saving and insurance.

[5] Starr-McCluer (1996) finds the opposite, that US households covered by health insurance save more than uninsured households. Hsu (2013) explains this finding with institutional factors such as safety nets and employer-provided insurance.

[6] Skinner (1988), Kuehlwein (1991), Guiso et al. (1992) and Parker (1999) also find little to no evidence of precautionary saving and, accordingly, little to no evidence of prudence. Lee and Sawada (2007) argue that Dynan's low estimates for relative prudence are due to an omitted-variable bias caused by the lack of liquidity constraints when deriving the Euler equation. Carroll (2001), Ludvigson and Paxson (2001) and Feigenbaum (2005) question the way the Euler equations are approximated for estimation in this literature. Fulford (2015) finds that households are mostly concerned with expenditure shocks instead of income risk.

decision-makers may differ in the portfolio of instruments they use to respond to income risk. In our setting, precautionary saving is fairly robust to substitution effects but precautionary self-protection and self-insurance are quite susceptible. This makes them less suited to infer precautionary preferences even though, at a qualitative level, they are subject to the same trade-offs as precautionary saving.

We proceed as follows. Section 2 introduces preferences, instruments and ordering relations for comparative statics. Section 3 analyzes precautionary motives when decision-makers use a single instrument. Section 4 covers interaction effects between instruments. Section 5 presents a detailed numerical analysis of precautionary motives. Section 6 discusses the role of instrument interaction for the inference of preference parameters. Section 7 concludes.

2 Preferences, instruments and ordering relations

We consider a decision-maker (DM) who lives for two periods. Her intertemporal consumption stream $(c_1, \widetilde{c}_2)$ consists of certain consumption c_1 in the first period and risky consumption $\widetilde{c}_2$ in the second period. The tilde indicates random variables. Preferences over consumption are represented by the following recursive utility (RU) objective,

$$u(c_1) + \beta u\left(\psi^{-1}\left(\mathbb{E}\psi(\widetilde{c}_2)\right)\right), \tag{1}$$

see Kreps and Porteus (1978) and Selden (1978). In this representation, u measures the DM's preference to smooth consumption over time, β is her utility discount factor, ψ measures risk preferences, and $\mathbb{E}$ is the expectation operator.[7] Both u and ψ are assumed to be strictly increasing and concave for now. When convenient, we denote the certainty equivalent of future consumption by

$$CE(\widetilde{c}_2) \equiv \psi^{-1}\left(\mathbb{E}\psi\left(\widetilde{c}_2\right)\right).$$

The DM receives a certain amount of income w_1 in the first period and w_2 in the second period. Consumption in the second period is risky due to the possibility of a monetary loss of size L that occurs with probability $p \in (0,1)$. We denote this loss risk $\widetilde{\ell}$. Besides the loss risk, an additional source of uncertainty in the second period is income risk. We model it as an additive zero-mean background risk $\widetilde{\varepsilon}$ with support $[\underline{\varepsilon}, \overline{\varepsilon}]$. In the presence of the income risk, second-period income is given by $\widetilde{w}_2 = w_2 + \widetilde{\varepsilon}$ instead of w_2. Assuming $\widetilde{\varepsilon}$ with mean zero allows us to focus on the pure risk effects on behavior. For tractability, we consider the loss risk and the income risk to be independent.

We consider three instruments in this paper that allow the DM to modify her intertemporal consumption stream. We introduce them in the following definition.

[7] A well-known special case is Epstein and Zin's (1991) specification with iso-elastic u and ψ functions.

Definition 1 (Instruments)**.**

- *Saving* s transfers income from the first to the second period at gross interest rate R.
- *Self-protection* is an upfront investment x that reduces the probability of loss to $p(x)$.
- *Self-insurance* is an upfront investment y that reduces the severity of loss to $L(y)$.

Each instrument involves an upfront cost, which reduces consumption in the first period, at the benefit of higher expected consumption in the second period. The instruments differ in the way they induce this increase in expected consumption. Saving raises consumption in each state in the second period, thereby providing a buffer against uncertainty. Self-protection and self-insurance instead affect the loss risk directly (Ehrlich and Becker, 1972). Self-protection reduces the expected loss in the second period by lowering the probability of loss without affecting its size. Self-insurance reduces the expected loss by lowering the magnitude of loss without altering its likelihood. Courbage et al. (2013) and the papers cited therein provide many specific examples of self-protection and self-insurance activities. In practice, DMs invest in safety to mitigate property and liability risks arising from vehicle and home ownership. They purchase insurance to reduce retained losses, which our definition of self-insurance can accommodate as well. While the distinction between self-protection and self-insurance may appear stylized, it has helped uncover several differences in their comparative statics, most notably when it comes to risk aversion (e.g., Dionne and Eeckhoudt, 1985).

To gain intuition and explain some of the differences between saving, self-protection and self-insurance, we compare how they affect the risk exposure in the second period. We look at the first three moments of second-period consumption and write σ and sk as shorthand for the standard deviation and the skewness of a random variable. Risk preferences are not moment preferences but such a comparison can still provide useful intuition. The following remark summarizes, see Online Appendix C.1 for a proof.

Remark 1 (Moment Effects of Instruments)**.**

- *Saving increases* $\mathbb{E}\widetilde{c}_2$*, but leaves* $\sigma(\widetilde{c}_2)$ *and* $sk(\widetilde{c}_2)$ *unaffected.*
- *Self-protection increases* $\mathbb{E}\widetilde{c}_2$*; it reduces* $\sigma(\widetilde{c}_2)$ *if and only if* $p(x) < 0.5$*. For* $\sigma(\widetilde{\varepsilon})sk(\widetilde{\varepsilon}) > -\frac{2}{3}L(y)$*, there is a threshold* p_1 *such that self-protection increases* $sk(\widetilde{c}_2)$ *if* $p(x) < p_1$*.*
- *Self-insurance increases* $\mathbb{E}\widetilde{c}_2$ *and reduces* $\sigma(\widetilde{c}_2)$*; it increases* $sk(\widetilde{c}_2)$ *if and only if* $p(x) < 0.5(1 + \sigma(\widetilde{\varepsilon})sk(\widetilde{\varepsilon})/L(y))$*.*

For optimization, we use methods of monotone comparative statics (e.g., Nocetti, 2016; Wang and Li, 2015, 2016; Wang et al., 2015; Wong, 2016). This approach overcomes the narrow focus on interior solutions and unique maximizers, which often requires additional restrictions on the primitives of the model to ensure global concavity of the objective function. In the absence of such subordinate assumptions, optimal decisions are not necessarily singletons but may be set-valued. To compare objective functions, we use Quah and Strulovici's (2009) so-called *interval-dominance order*.

Definition 2 (Interval Dominance Order)**.** Let f and g be two real-valued functions defined on $Z \subset \mathbb{R}$. We say that g dominates f by the *interval dominance order*, denoted $g \succeq_I f$, if

$$f(z'') - f(z') \geq (>)\ 0 \qquad \Longrightarrow \qquad g(z'') - g(z') \geq (>)\ 0$$

holds for z'' and z' such that $z'' > z'$ and $f(z'') \geq f(z)$ for all z in the interval $[z', z''] \equiv \{z \in Z : z' \leq z \leq z''\}$.

Ranking objective functions by the interval-dominance order is less restrictive than alternative ordering concepts but still allows for simple proofs.[8] Quah and Strulovici's (2009) Proposition 1 characterizes the interval-dominance order for continuous and piecewise monotone functions. A function $f : Z \to \mathbb{R}$ is *regular* if $\arg\max_{z \in [z', z'']} f(z)$ is nonempty for any points z' and z'' with $z'' > z'$. For later reference, we state their comparative static result.

Theorem 1 (Quah and Strulovici, 2009)**.** *Suppose that f and g are real-valued functions defined on $Z \subset \mathbb{R}$ and $g \succeq_I f$. Then,*

$$\arg\max_{z \in J}\ g(z) \geq_S \arg\max_{z \in J} f(z) \quad \textit{for any interval } J \textit{ of } Z. \tag{2}$$

Furthermore, if (2) *holds and g is regular, then $g \succeq_I f$.*

The comparison between the maximizers of g and f in condition (2) is stated in terms of the *strong set order*, denoted by $\geq_S$. For two subsets Z' and Z'' of $\mathbb{R}$, Z'' is larger than Z' in the strong set order if, for any $z'' \in Z''$ and $z' \in Z'$, we have $\max\{z'', z'\} \in Z''$ and $\min\{z'', z'\} \in Z'$. If both sets are singletons, $Z'' = \{z''\}$ and $Z' = \{z'\}$, then $Z'' \geq_S Z'$ collapses to the usual $z'' \geq z'$. More generally, if both sets contain their largest and smallest elements, then $Z'' \geq_S Z'$ implies $\max Z'' \geq \max Z'$ and $\min Z'' \geq \min Z'$.

3 Precautionary behavior with a single instrument

3.1 Saving

We first investigate precautionary saving. In the benchmark situation without income risk, the DM maximizes objective function

$$U(s; 0) = u(w_1 - s) + \beta u\left(CE(w_2 + sR + \widetilde{\ell})\right)$$

over $s \in [-(w_2 - L)/R, w_1]$. In the presence of income risk, she maximizes

$$U(s; \widetilde{\varepsilon}) = u(w_1 - s) + \beta u\left(CE(\widetilde{w}_2 + sR + \widetilde{\ell})\right),$$

[8] Increasing differences and the single-crossing condition each imply interval dominance. Quah and Strulovici (2009) provide an explicit example to show that the interval-dominance order is less restrictive than the single-crossing property. Quah and Strulovici (2007) and Sobel (2019) compare all these ordering relations.

over $s \in [-(w_2 + \underline{\varepsilon} - L)/R, w_1]$. Saving is the DM's only instrument for now, and therefore the loss risk has a fixed probability and severity.

We call the DM prudent if the maximizers of $U(s; \widetilde{\varepsilon})$ are larger than the maximizers of $U(s; 0)$ in the strong set order, and imprudent if the reverse ordering holds. A prudent DM is said to engage in precautionary saving because income risk raises her optimal saving choice(s) in the sense of the strong set order. Proposition 1 presents sufficient conditions.

Proposition 1 (Precautionary Saving)**.** *Consider the effect of income risk on optimal saving. The DM is:*

(i) Prudent if ψ' is convex and u is more concave than ψ;

(ii) Imprudent if ψ' is concave and u is less concave than ψ.

We provide a proof in Appendix A.1. The stated conditions allow us to rank $U(s; \widetilde{\varepsilon})$ and $U(s; 0)$ by the interval dominance order and then apply Theorem 1. The conditions in statement (i) are well-known in the consumption-saving literature under recursive utility (see Kimball and Weil, 2009; Gollier, 2001; Wang and Li, 2016). Proposition 1 shows that they also apply to situations where income risk represents an additional source of uncertainty because a loss risk is already present. The ordering of the maximizers can be either way, so that both prudent and imprudent behavior are possible. Table 1 provides an overview.

	$\psi''' \geq 0$	$\psi''' \leq 0$
u more concave than ψ	prudence	indeterminate
u less concave than ψ	indeterminate	imprudence

Table 1: Sufficient conditions for prudence and imprudence under RU

The conditions combine Kimball's (1990) prudence condition, $\psi''' \geq 0$, from the additively separable expected utility model, with the relative curvature of u and ψ, a measure of the DM's attitude towards the timing of uncertainty resolution. If u is more (less) concave than ψ, the DM prefers a late (early) resolution of uncertainty, see Proposition 77 in Gollier (2001). Using real incentives, von Gaudecker et al. (2011) find the preference for early versus late resolution of uncertainty evenly split in a representative sample of the Dutch population.

For intuition, we analyze how income risk affects the marginal benefit of saving under RU,

$$\beta R \frac{u'(CE(\widetilde{c}_2))}{\psi'(CE(\widetilde{c}_2))} \mathbb{E}\psi'(\widetilde{c}_2).$$

There are two channels, a certainty equivalent (CE) channel and a marginal expected utility (MEU) channel, see Bostian and Heinzel (2020). With a concave ψ, income risk lowers CE in $u'(CE(\widetilde{c}_2))$, which raises the marginal value of saving for reasons of consumption smoothing.

At the same time, income risk changes the sensitivity of CE with respect to saving, which is given by $\mathrm{d}CE/\mathrm{d}s = \mathbb{E}\psi'(\widetilde{c}_2)/\psi'(CE(\widetilde{c}_2))$. If ψ' is convex, saving raises CE by more when income risk is present and the numerator of $\mathrm{d}CE/\mathrm{d}s$ increases. This represents a positive MEU effect. At the same time, income risk raises $\psi'(CE(\widetilde{c}_2))$, the denominator of $\mathrm{d}CE/\mathrm{d}s$, which makes CE less sensitive to saving. If u is more concave than ψ, this negative effect is outweighed by the positive consumption smoothing effect. In this case, the net effect of the CE channel is positive.

The literature on precautionary saving under RU has identified another condition for prudence (see Kimball and Weil, 2009). If income risk is the only source of uncertainty and ψ exhibits decreasing absolute risk aversion (DARA), the DM accumulates precautionary saving under RU without any restrictions on her felicity function u other than concavity. If a loss risk is already present, DARA of ψ is no longer strong enough and we need to impose a more restrictive assumption, namely constant absolute risk aversion (CARA).

Remark 2. *The DM is prudent for any concave felicity function u if ψ has CARA.*

We provide a proof in Appendix A.2. Intuitively, if ψ satisfies CARA, the additive income risk $\widetilde{\varepsilon}$ is multiplicatively separable both in terms of expected utility and expected marginal utility. As a result, it does not affect the sensitivity of CE with respect to saving. In technical terms, the ratio $\mathbb{E}\psi'(\widetilde{c}_2)/\psi'(CE(\widetilde{c}_2))$ is unaffected by income risk and its only effect is a smaller CE, which stimulates saving to smooth consumption.

3.2 Self-protection and self-insurance

We now turn to precautionary self-protection and precautionary self-insurance. For self-protection, the DM's objective function is given by

$$U(x;0) = u(w_1 - x) + \beta u\left(CE(w_2 + \widetilde{\ell})\right)$$

in the absence of income risk and by

$$U(x;\widetilde{\varepsilon}) = u(w_1 - x) + \beta u\left(CE(\widetilde{w}_2 + \widetilde{\ell})\right),$$

in the presence of income risk. Both are maximized over $x \in [0, w_1]$. The loss risk has a binary distribution. A loss of L occurs with probability $p(x)$ whereas no loss occurs with probability $(1-p(x))$. Using the same terminology as before, we call a DM prudent (imprudent) if income risk increases (decreases) self-protection in the strong set order.

For self-insurance, the DM's objective function is given by

$$U(y;0) = u(w_1 - y) + \beta u\left(CE(w_2 + \widetilde{\ell})\right)$$

in the absence of income risk and by

$$U(y;\widetilde{\varepsilon}) = u(w_1 - y) + \beta u\left(CE(\widetilde{w}_2 + \widetilde{\ell})\right)$$

in the presence of income risk with $y \in [0, w_1]$. Now a loss of $L(y)$ occurs with probability p whereas no loss occurs with probability $(1-p)$. Prudence and imprudence are defined as before. For both instruments, we find the following result.

Proposition 2 (Precautionary Self-Protection and Self-Insurance)**.** *Consider the effect of income risk on optimal self-protection or optimal self-insurance. The DM is:*

(i) Prudent if ψ' is convex and u is more concave than ψ;

(ii) Imprudent if ψ' is concave and u is less concave than ψ.

We provide a proof in Appendix A.3. The intuition resembles the case of precautionary saving. Income risk affects the marginal benefit of either self-protection or self-insurance via two channels under RU. It lowers CE, and the relative concavity of u and ψ allows us to conclude whether this decrease in CE affects the marginal benefit positively or negatively. Furthermore, if ψ' is convex, an increase in either self-protection or self-insurance raises expected marginal utility in the presence of income risk by more than in its absence, whereas the reverse is true if ψ' is concave. Accordingly, Table 1 extends to the instruments of self-protection and self-insurance.

Proposition 2 generalizes previous findings on precautionary self-protection to RU (see Eeckhoudt et al., 2012; Courbage and Rey, 2012; Wang and Li, 2015). Indeed, if $u = \psi$, we obtain the additively separable expected utility model as a special case, and condition (i) simplifies to ψ' being convex. Precautionary self-insurance has not been considered explicitly in the literature yet. Eeckhoudt and Kimball (1992) find conditions for an uninsurable background risk to raise insurance demand against a foreground risk but they consider a single period. Wang et al. (2015) use additively separable expected utility preferences. Their model of precautionary paying covers self-insurance activities. They show that the conditions for precautionary self-protection under expected utility also ensure precautionary paying, see their Proposition 3.2. Proposition 2 extends the analysis of self-insurance to RU.

As in the case of saving, we can specify a restriction on ψ alone that guarantees prudent behavior. Remark 2 also holds for self-protection and self-insurance. If ψ exhibits CARA, an additive income risk $\widetilde{\varepsilon}$ is multiplicatively separable in terms of expected utility and expected marginal utility. In this case, income risk raises self-protection or self-insurance to compensate for the lower CE and smooth consumption.

3.3 Costly risk reduction: A unifying approach

The similarity between saving, self-protection and self-insurance motivates the development of a unifying approach, that contains the previous results as special cases. As a part of this generalization, we consider the practically more relevant changes in income risk from "risk" to "greater risk," instead of the restrictive comparison between "no risk" and "risk." As another extension, we examine both risk averters and risk lovers. We first provide some background on Nth-degree risk increases. Consider two random variables with support contained in $[\underline{z}, \overline{z}]$ and cumulative distribution functions F and G. We set $F^{(1)}(z) \equiv F(z)$ and define recursively $F^{(i)}(z) = \int_a^z F^{(i-1)}(t)\,\mathrm{d}t$ for integers $i \geq 2$ and likewise for G.

Definition 3 (Ekern, 1980)**.** *G has more Nth-degree risk than F if*

(i) $F^{(N)}(z) \leq G^{(N)}(z)$ for all $z \in [\underline{z}, \overline{z}]$,

(ii) $F^{(i)}(\overline{z}) = G^{(i)}(\overline{z})$ for all $i \in \{1, \ldots, N\}$.

Condition (ii) preserves the first $(N-1)$ moments when increasing Nth-degree risk while condition (i) implies an increase in the Nth moment, sign adjusted by $(-1)^N$. Well-known special cases are first-order stochastic dominance for $N = 1$, an increase in risk for $N = 2$ (Rothschild and Stiglitz, 1970), an increase in downside risk for $N = 3$ (Menezes et al., 1980), and an increase in outer risk for $N = 4$ (Menezes and Wang, 2005). If G has more Nth-degree risk than F, we write $F \succeq_N G$. This ordering relation is useful via its link to expected utility. We use the notation $\psi^{(N)}(c)$ for $\mathrm{d}^N\psi(c)/\mathrm{d}c^N$ and formulate a familiar result.

Theorem 2 (Ekern, 1980)**.** *The following statements are equivalent:*

(i) G has more Nth-degree risk than F,

(ii) $\int_{\underline{z}}^{\overline{z}} \psi(z)\,\mathrm{d}F(z) \geq \int_{\underline{z}}^{\overline{z}} \psi(z)\,\mathrm{d}G(z)$, for all functions ψ with $(-1)^{N+1}\psi^{(N)} \geq 0$.

According to Theorem 2, Nth-degree risk increases are precisely the risk changes which are disliked by all DMs whose utility function satisfies the sign condition in (ii), see also Denuit et al. (1999) and Jouini et al. (2013). For this reason, Ekern (1980) calls these DMs Nth-degree risk-averse. Special cases of Nth-degree risk aversion include non-satiation ($\psi' \geq 0$, $N = 1$), risk aversion ($\psi'' \leq 0$, $N = 2$), downside risk aversion ($\psi''' \geq 0$, $N = 3$) and temperance ($\psi^{(4)} \leq 0$, $N = 4$). Similarly, we define DMs to be Nth-degree risk-loving if their utility function satisfies $(-1)^{N+1}\psi^{(N)} \leq 0$. Special cases include risk loving ($\psi'' \geq 0$, $N = 2$), downside risk loving ($\psi''' \leq 0$, $N = 3$) and intemperance ($\psi^{(4)} \geq 0$, $N = 4$). We denote by $\Psi_N^{r.a.}$ the collection of all utility functions that satisfy $(-1)^{N+1}\,\psi^{(N)} \geq 0$ and are thus Nth-degree risk-averse, and by $\Psi_N^{r.l.}$ the collection of all utility functions that satisfy $(-1)^{N+1}\psi^{(N)} \leq 0$ and are thus Nth-degree risk-loving.

To connect this to our previous analysis, consider a DM who faces two independent risks in the second period, an exogenous income risk $\widetilde{\varepsilon}$ and an endogenous loss risk $\widetilde{\ell}$ with cumulative distribution function $F(\ell; a)$. We parameterize the risk-reducing activity by its upfront cost a in the first period. This cost reduces the Nth-degree riskiness of $\widetilde{\ell}$ in the second period, $F(\ell; a'') \succeq_N F(\ell; a')$ for $a'' \geq a'$. The activity level a is contained in $[\underline{a}, \overline{a}]$ and we focus on Nth-degree risk averters.[9] Saving, self-protection and self-insurance are special cases of risk-reducing activities for $N = 1$ because they increase second-period consumption in the sense of first-order stochastic dominance.

We may now wonder how the riskiness of the exogenous income risk $\widetilde{\varepsilon}$ affects the DM's behavior towards the endogenous risk. If $\widetilde{\varepsilon}''$ has more Mth-degree risk than $\widetilde{\varepsilon}'$, we would like to compare the solution of

$$\max_{a\in[\underline{a},\overline{a}]} U(a;\widetilde{\varepsilon}') = u(w_1 - c(a)) + \beta u\left(CE(w_2 + \widetilde{\varepsilon}' + \widetilde{\ell})\right)$$

to the solution of

$$\max_{a\in[\underline{a},\overline{a}]} U(a;\widetilde{\varepsilon}'') = u(w_1 - c(a)) + \beta u\left(CE(w_2 + \widetilde{\varepsilon}'' + \widetilde{\ell})\right).$$

The following proposition summarizes our findings.

Proposition 3 (Precautionary Risk Reduction)**.** *Consider a DM with $\psi \in \Psi_N^{r.a.}$ who engages in costly Nth-degree risk reduction. For $\psi \in \Psi_M^{r.a.}$, an Mth-degree risk increase of an independent income risk:*

(i) Raises optimal risk reduction if $\psi \in \Psi_{M+N}^{r.a.}$ and u is more concave than ψ;

(ii) Lowers optimal risk reduction if $\psi \in \Psi_{M+N}^{r.l.}$ and u is less concave than ψ.

For $\psi \in \Psi_M^{r.l.}$, an Mth-degree risk increase of an independent income risk:

(iii) Lowers optimal risk reduction if $\psi \in \Psi_{M+N}^{r.l.}$ and u is more concave than ψ;

(iv) Raises optimal risk reduction if $\psi \in \Psi_{M+N}^{r.a.}$ and u is less concave than ψ.

We provide a proof in Appendix A.4. We recoup Propositions 1 and 2 as special cases from Proposition 3(i) and (ii) by setting $N = 1$ and $M = 2$. Obviously, results (iii) and (iv) require $M \neq N$ so that Mth-degree risk loving does not conflict with Nth-degree risk aversion. The conditions in Proposition 3 allow us to rank $U(a;\widetilde{\varepsilon}')$ and $U(a;\widetilde{\varepsilon}'')$ by the interval dominance order and Theorem 1 then establishes the ranking of the maximizers in the strong set order. Intuitively, if the DM is risk-averse at orders N, M and $M + N$, and u is more concave than ψ, then the Mth-degree increase in income risk raises the marginal value of reducing the Nth-degree riskiness of the endogenous risk. Mth-degree risk aversion implies a lower CE in response to the Mth-degree risk increase. This reduction in CE has a positive effect on the value of Nth-degree risk reduction if u is more concave than ψ due to the CE channel. $(M + N)$th-degree risk aversion ensures that Mth-degree risk increases and Nth-degree risk increases are mutually aggravating (see Ebert et al., 2018), which is the analog

[9] Due to the upfront cost, an Nth-degree risk lover would always choose the lowest possible level of the activity $a = \underline{a}$ because she does not value Nth-degree risk reduction. Then, all comparative statics are trivial.

of the positive MEU channel. Under the stated assumptions, both channels are aligned and optimal risk reduction increases. Table 2 provides an overview in compact form.

	$\psi \in \Psi_M^{r.a.}$		$\psi \in \Psi_M^{r.l.}$	
	$\psi \in \Psi_{M+N}^{r.a.}$	$\psi \in \Psi_{M+N}^{r.l.}$	$\psi \in \Psi_{M+N}^{r.a.}$	$\psi \in \Psi_{M+N}^{r.l.}$
u more concave than ψ	increase	indet.	indet.	decrease
u less concave than ψ	indet.	decrease	increase	indet.

Table 2: Effect of an Mth-degree increase in income risk on Nth-degree risk reduction for Nth-degree risk-averse DMs. The notation $\psi \in \Psi_M^{r.a.}$ is shorthand for Mth-degree risk aversion, $(-1)^{M+1}\psi^{(M)} \geq 0$, and $\psi \in \Psi_M^{r.l.}$ is shorthand for Mth-degree risk loving, $(-1)^{M+1}\psi^{(M)} \leq 0$.

Risk lovers have been receiving increasing attention in recent years (see Crainich et al., 2013; Jindapon, 2013; Jindapon and Whaley, 2015), which is why we emphasize results (*iii*) and (*iv*). In typical experiments on higher-order risk attitudes, Mth-degree risk loving preferences always play some role and should not be ignored (see Trautmann and van de Kuilen, 2018). In Table 2, the condition on the DM's attitude towards the timing of uncertainty resolution switches when going from the left panel (for Mth-degree risk aversion) to the right panel (for Mth-degree risk loving). This is because Mth-degree risk lovers appreciate the Mth-degree risk increase in income risk, which then leads to a *higher* CE, not a lower one. To align the CE channel with the MEU channel, we then need to reverse the assumption about the relative concavity of u and ψ.

Proposition 3 extends Wang and Li's (2016) result on precautionary saving under RU to general forms of risk reduction behavior. They focus exclusively on risk averters whereas we consider risk lovers as well. Proposition 3 also extends Wang et al.'s (2015) results on precautionary paying in the additively separable expected utility model. They consider a possibly non-financial background risk, an extension we could readily provide, and do not consider risk lovers. Behavior in their model reduces riskiness in the sense of Nth-order stochastic dominance, which is more general than Ekern (1980) risk effects. Our focus on Nth-degree risk brings out clearly how the orders associated with endogenous and exogenous risk changes correspond to the preference conditions. If individuals are mixed risk-averse (Caballé and Pomansky, 1996), Proposition 3(i) predicts an increase in risk reduction as long as u is more concave than ψ. Mixed risk aversion is a consistency requirement of "combining good with bad" and satisfied in many common classes of utility functions (Brockett and Golden, 1987). While evidence exists in support of it (Deck and Schlesinger, 2014), there are recent findings to the contrary (Bleichrodt and van Bruggen, 2021). Proposition 3 presents all combinations of assumptions that admit unambiguous comparative statics.

The consideration of risk averters and risk lovers also highlights the need to distinguish the DM's behavioral response to the Mth-degree increase in income risk from its welfare effect. All DMs in Proposition 3(i) and (ii) are worse off due to the risk change because $\psi \in \Psi_M^{r.a.}$ but some of them increase the level of risk reduction while others decrease it. Similarly, all DMs in Proposition 3(iii) and (iv) are better off due to the risk change because $\psi \in \Psi_M^{r.l.}$, but the optimal level of risk reduction may increase or decrease. So the welfare effect of the risk change contains no information about the direction of the associated behavioral response. On the flip-side, among those DMs who react by increasing Nth-degree risk reduction, some are made worse off by the increase in Mth-degree risk while others are made better off.

4 Instrument interaction

4.1 Interaction between specific instruments

We now proceed to situations where the DM can use more than one instrument to optimize intertemporal consumption and react to income risk. We first focus on the specific instruments outlined in Definition 1. If saving and self-protection are both available to the DM, her objective function is given by

$$U(s,x) = u(w_1 - s - x) + \beta u\left(CE(w_2 + sR + \widetilde{\ell})\right),$$

with

$$\psi\left(CE(w_2 + sR + \widetilde{\ell})\right) = p(x)\psi(w_2 + sR - L) + (1 - p(x))\psi(w_2 + sR).$$

If she uses saving and self-insurance, her objective function is

$$U(s,y) = u(w_1 - s - y) + \beta u\left(CE(w_2 + sR + \widetilde{\ell})\right),$$

with

$$\psi\left(CE(w_2 + sR + \widetilde{\ell})\right) = p\psi(w_2 + sR - L(y)) + (1 - p)\psi(w_2 + sR).$$

If she uses self-protection and self-insurance, her objective function is

$$U(x,y) = u(w_1 - x - y) + \beta u\left(CE(w_2 + \widetilde{\ell})\right),$$

with

$$\psi\left(CE(w_2 + \widetilde{\ell})\right) = p(x)\psi(w_2 - L(y)) + (1 - p(x))\psi(w_2).$$

In any of these cases, the instruments interact in a nontrivial way, which is the subject of our next proposition.

Proposition 4 (Interaction Effects)**.** *Let u and ψ be strictly increasing and concave. If u is more concave than ψ, then any pair out of saving, self-protection and self-insurance exhibits Edgeworth-Pareto substitution in the sense of Samuelson (1974).*

A proof is given in Appendix A.5. If u is concave, the marginal cost of an instrument increases in the use of the other instrument because both instruments compete for resources in the first period. In the second period, the marginal benefit of an instrument decreases in the use of the other instrument because the marginal value of increasing CE is higher when CE is low. Consequently, a substitution effect arises between any pair of instruments.

Proposition 4 extends a number of results to RU. Dionne and Eeckhoudt (1984) study the Hicksian demand for saving and insurance in an expected utility model with non-separable utility. Under decreasing temporal risk aversion, saving is then a substitute for insurance.[10] Similarly, Menegatti and Rebessi (2011), Hofmann and Peter (2016) and Peter (2017) find a substitution effect between saving and self-protection or between saving and self-insurance. They focus on the additively separable expected utility model.

If u is more concave than ψ, the DM prefers a late resolution of uncertainty. In this case, $\psi''' \geq 0$ ensures prudence in the single-instrument cases, see Propositions 1(i) and 2(i). Income risk then exerts a positive precautionary effect on each instrument. Edgeworth-Pareto substitution between the instruments introduces, in addition, conflicting substitution effects. As a result, in Nocetti's (2013) words, the instruments are neither income risk complements nor income risk substitutes because net effects are ambiguous. We conclude that the prevalent focus on single decision variables in the literature is by no means a simplifying assumption. Instead, it is the very reason why we obtain definitive results like Proposition 3.

4.2 Interaction in costly risk reduction

Proposition 4 applies to any pair of instruments, which points to a general mechanism. Let us decompose the loss risk into two independent components, $\widetilde{\ell} = \widetilde{\ell}^1 + \widetilde{\ell}^2$, and let $\widetilde{\ell}^j$ be distributed according to the distribution function $F_j(\ell; a_j)$ for $j = 1, 2$. Consider two activities, similar to Section 3.3, that reduce the N_1th- and the N_2th-degree riskiness of second-period consumption against an upfront cost of a_1 and a_2 in the first period. N_jth-degree risk reduction implies that $F_j(\ell; a_j'') \succeq_{N_j} F(\ell; a_j')$ for $a_j'' \geq a_j'$. The DM's objective function is

$$U(a_1, a_2) = u(w_1 - a_1 - a_2) + \beta u\left(CE(w_2 + \widetilde{\ell})\right),$$

with

$$\psi\left(CE(w_2 + \widetilde{\ell})\right) = \int\int \psi(w_2 + \ell^1 + \ell^2)\, \mathrm{d}F_1(\ell^1; a_1)\, \mathrm{d}F_2(\ell^2; a_2),$$

The next proposition examines the relationship between the two risk-reducing activities.

Proposition 5 (Interaction Effects)**.** *For a concave felicity function u, let $\psi \in \Psi_i^{r.a.}$ for $i = N_1, N_2, N_1 + N_2$. If u is more concave than ψ, then N_1th-degree risk reduction and N_2th-degree risk reduction are Edgeworth-Pareto substitutes in the sense of Samuelson (1974).*

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[10] Decreasing temporal risk aversion simplifies to decreasing absolute risk aversion with respect to second-period consumption in the additively separable expected utility model.

Appendix A.6 provides a proof. Proposition 4 is a special case of Proposition 5 for $N_1 = N_2 = 1$, which yields the assumptions that ψ be strictly increasing and concave. The two risk-reduction activities compete for resources in the first period so that an increase in either activity raises the marginal cost of engaging in the other activity. In the second period, an increase in either activity raises CE, which is more valuable when CE is low than when it is high, that is, when the other activity is at a lower level. This represents a negative CE channel because u is more concave than ψ. Furthermore, due to $(N_1 + N_2)$th-degree risk aversion, N_1th-degree risk increases and N_2th-degree risk increases are mutually aggravating (Ebert et al., 2018). Hence, the marginal value of either activity is lower the higher the level of the other activity, corresponding to a negative MEU channel. In conjunction, a substitution effect arises between the two activities.

The indeterminacy mentioned after Proposition 4 extends to general risk-reduction activities. If u is more concave than ψ, a mixed risk-averse DM experiences a positive precautionary effect on each instrument in response to greater income risk, see Proposition 3(i). Due to Edgeworth-Pareto substitution between instruments, each positive precautionary effect is flanked by a negative substitution effect, and net effects are ambiguous. In Nocetti's (2013) terminology, N_1th-degree risk reduction and N_2th-degree risk reduction are neither Mth-degree risk complements nor Mth-degree risk substitutes for income risk.

Instrument interaction persists when considering more than two instruments. Say a DM uses saving, self-protection and self-insurance all at a time. If u is more concave than ψ and $\psi''' \geq 0$, income risk exerts a positive precautionary effect on each instrument, which is now flanked by two negative substitution effects, one from each of the other instruments. Net effects are then indeterminate *a fortiori*.

5 Numerical analysis

5.1 Preliminaries and parameters

Our propositions treat directional changes and do not inform about magnitudes. We calibrate the model to measure the extent of precautionary reactions to income risk. This sheds further light on Propositions 1 to 3 by comparing precautionary responses across instruments and assessing the value of each instrument for the DM. We also quantify interaction effects in situations with multiple instruments, see Propositions 4 and 5. Finally, we look at scenarios in which, contrary to Propositions 1 to 5, the CE channel and the MEU channel are not aligned. While numerical results depend on functional form assumptions and parameter values, they help shed light on the potential significance of theoretical trade-offs.

To implement RU preferences as in (1), we use Epstein and Zin's (1991) specification with iso-elastic u and ψ functions. We set

$$u(c) = \begin{cases} c^{1-\alpha}/(1-\alpha) & \text{if } \alpha \neq 1, \\ \ln(c) & \text{if } \alpha = 1, \end{cases} \quad \text{and} \quad \psi(c) = \begin{cases} c^{1-\gamma}/(1-\gamma) & \text{if } \gamma \neq 1, \\ \ln(c) & \text{if } \gamma = 1. \end{cases}$$

Parameter α is the resistance to intertemporal substitution of consumption, equal to the inverse of the elasticity of intertemporal substitution (EIS), and parameter γ measures relative risk aversion. For both parameters, we consider a value range from 1 to 5 and set $\alpha = 3$ and $\gamma = 2$ in the base case.[11] We are thus in the situation of Propositions 1(i) and 2(i) because ψ' is convex and u is more concave than ψ. We set $\beta = 1$ for simplicity and briefly discuss its effect on precautionary behavior at the end of Section 5.2.

For the instruments we set the gross return on saving to $R = 1$ and specify self-protection and self-insurance as

$$p(x) = p_0\, e^{-\mu x} \quad \text{ and } \quad L(y) = L_0\, e^{-\nu y},$$

where $p_0 \in (0, 1)$ is the baseline probability of loss, L_0 the baseline severity of loss, and μ and ν are positive efficiency parameters. Briys et al. (1991) use a negative exponential specification for risky self-insurance with uncertain effectiveness (see also Li and Peter, 2021), and Barro (2015) uses this functional form for self-protection in the context of optimal environmental investment. We set $\mu = 0.0015355$ and $\nu = 0.0012866$ in the base case. As stated in Courbage et al.'s (2013) survey article, the empirical literature on prevention is thin so there is no descriptive guidance on the size of these parameters.[12]

For the remaining economic parameters, we set $w_1 = w_2 = \$50,000$, which corresponds roughly to the annual median income for individuals 25 years and older with a bachelor's degree in the US.[13] We set $p_0 = 10\%$ and $L_0 = \$10,000$, resulting in an expected unmitigated loss of \$1,000 or 2% of annual income. While we have no particular risk exposure in mind, examples include physical damage and liability risks arising from home and vehicle ownership, uncovered healthcare costs, unanticipated maintenance or repair costs, etc.

The background risk $\widetilde{\varepsilon}$ on future income is the root cause of precautionary behavior. We focus on increases in riskiness and downside riskiness by setting $\mathbb{E}\widetilde{\varepsilon} = 0$, like in our theoretical analysis. We use binary lotteries, which are fully characterized by their first three moments (see Ebert, 2015). Empirically, economists have analyzed annual log earnings growth to study the cross-sectional and dynamic properties of income risk. Based on a large panel data set of tax form W2 (Wage and Tax Statement) filings in the US, Guvenen et al. (2021) find substantial deviations from lognormality with strong negative skewness and high kurtosis.

[11] Gollier (2001) suggests that relative risk aversion ranges from 1 to 4. Meyer and Meyer (2005) adjust reported values of relative risk aversion to account for different ways its argument is measured (i.e., consumption, wealth, income). Most adjusted values are between 1 and 5. For EIS, Havránek (2015) finds strong selective reporting in the literature. He states a corrected mean of micro estimates for asset holders around 0.3-0.4, corresponding to α values between 2.5 and 3.33. Thimme (2017) concludes that, for representative agents who consume a single nondurable consumption good, EIS should clearly be below unity.

[12] Our parameter choice reflects a compromise and ensures that all technologies are in use in the base case. If μ or ν is high, a small investment suffices to reduce the probability or severity of loss considerably, which leaves little room for precautionary behavior. If μ or ν is low, the technologies are ineffective and will not be used or be dominated by other technologies. Appendix B discusses how precautionary instrument use depends on the respective technology parameters.

[13] See the Bureau of Labor Statistics, `www.bls.gov/emp/chart-unemployment-earnings-education.htm`.

Recently, De Nardi et al. (2020) analyze annual household level after-tax earnings growth for the PSID (Panel Study of Income Dynamics) data. Their focus on household disposable income attenuates the magnitudes of the higher-order moments in Guvenen et al. (2021) due to intra-family risk sharing (Blundell et al., 2016), but lognormality is still strongly rejected. In De Nardi et al. (2020), the standard deviation of annual log earnings growth ranges from 0.25 to 0.6 with most values below 0.4 and skewness between -2 and 0 (see the bottom panel of their Figure 1).

For riskiness, we specify $\widetilde{\varepsilon}$ as a 50-50 chance of realizing a gain or loss of ε in annual income, that is, $\widetilde{\varepsilon} = [0.5, -\varepsilon;\ 0.5, \varepsilon]$. We vary ε in increments of \$5,000 between \$0 for riskless income and \$20,000 for an income risk of 40% of annual income. This yields a standard deviation of annual log earnings growth between 0 and 0.42 while skewness is uniformly zero. For downside risk, we use the construction in Ebert's (2015) Proposition 1 to obtain skewed risks with a mean of zero, a standard deviation of 25% of annual income, and skewness ranging from 0 to -2 in decrements of 0.5 . We set $\widetilde{\varepsilon} = [q, \varepsilon_-;\ (1-q), \varepsilon_+]$ and solve for the unique $q \in (0,1)$, $\varepsilon_- < 0$ and $\varepsilon_+ > 0$ to generate the first three moments accordingly. Table 9 in Online Appendix C.2 provides the corresponding parameter values. The log earnings growth of these income risks has a standard deviation ranging from 0.26 to 0.36 and a skewness between 0 and -2. Table 3 summarizes all parameter choices for the base case.[14]

In our numerical set-up, each objective function has a unique interior maximizer in the single-instrument cases and when multiple instruments are available. To conduct welfare comparisons, we also report *smooth certainty-equivalent consumption* c_{sce}. We define it as the riskless time-invariant consumption stream (c_{sce}, c_{sce}) that yields a given level of RU. For instance, when the income risk is $\widetilde{\varepsilon}$ and the optimal level of saving is s^*, smooth certainty-equivalent consumption is implicitly given by

$$u\left(c_{sce}\right) + \beta u\left(c_{sce}\right) = U\left(s^*; \widetilde{\varepsilon}\right).$$

It is measured in dollars and can thus be compared across risk levels and instruments. Our measure of welfare is comparable to Wang et al.'s (2016) certainty-equivalent wealth.

5.2 Precaution with a single instrument

As a benchmark, we first consider the single-instrument cases from Section 3. We denote the optimal level of saving in the absence of income risk by $s^0 = \arg\max_s U(s; 0)$ and the optimal level of saving in the presence of income risk by $s^* = \arg\max_s U(s; \widetilde{\varepsilon})$. The amount of precautionary saving is then given by $s^\pi = s^* - s^0$ and the fraction of savings that are precautionary is s^π / s^*. These notations apply analogously to self-protection and self-insurance.

[14] The binary risk assumption understates the kurtosis of the log earnings growth. It ranges from 1 to 5 in our examples whereas De Nardi et al. (2020) find kurtosis up to 20 with many values around 10. We leave investigating how kurtosis affects our results for future research.

Parameter	Description	Value
Preference parameters		
α	Inverse of EIS	3
γ	Relative risk aversion	2
β	Utility discount factor	1
Instruments		
R	Gross return on saving	1
μ	Efficiency of self-protection	0.0015355
p_0	Baseline probability of loss	10%
ν	Efficiency of self-insurance	0.0012866
L_0	Baseline severity of loss	\$10,000
Economic parameters		
w_1	Income in first period	\$50,000
w_2	Income in second period	\$50,000
$\widetilde{\varepsilon}$	Income risk (*symmetric*)	$[0.5, -\varepsilon;\ 0.5, +\varepsilon]$, $\sigma(\widetilde{\varepsilon})/w_2 \in [0, 0.4]$
	Income risk (*skewed*)	$[q, \varepsilon_-;\ (1-q), \varepsilon_+]$, $sk(\widetilde{\varepsilon}) \in [-2, 0]$

Table 3: Parameter values for the base case.

Table 4 reports the results for symmetric income risks. As predicted by Propositions 1 and 2, income risk induces precautionary saving, self-protection and self-insurance because the level of each instrument is higher in the presence of income risk than in the absence of income risk. For each instrument, the precautionary component increases in income risk at an increasing rate. At high levels of income risk, precautionary saving accounts for more than 80% of total saving, and precautionary self-protection and self-insurance account for 40-55% of total instrument use. Saving shows by far the strongest precautionary response, exceeding that of self-protection and self-insurance by a factor of roughly 9 to 12.[15] Saving does not affect the loss risk directly so the expected loss is \$1,000 regardless of the size of the income risk. Self-protection and self-insurance mitigate the loss risk by reducing either its probability or its severity. Without income risk the DM faces an expected loss of \$527 in case of self-protection and of \$606 for self-insurance. Income risk induces precautionary behavior, which then lowers the expected loss as indicated by the change in $p(x^*)$ and $L(y^*)$.

[15] Precautionary self-protection and precautionary self-insurance are inverse U-shaped in the efficiency parameters μ and ν (see Appendix B). Even at their respective peak, the precautionary response of saving is still higher by a factor of 6.

	Saving			Loss risk		Moments of $\widetilde{c}_2$			
$\frac{\sigma(\widetilde{\varepsilon})}{w_2}$	s^*	s^π	s^π/s^*	p_0	L_0	$\mathbb{E}\widetilde{c}_2$	$\sigma(\widetilde{c}_2)$	$sk(\widetilde{c}_2)$	c_{sce}
0%	651	0	0%	10%	10,000	49,651	3,000	-2.67	49,392
10%	991	340	34%	10%	10,000	49,991	5,831	-0.36	49,137
20%	1,970	1,320	67%	10%	10,000	50,970	10,440	-0.06	48,390
30%	3,485	2,834	81%	10%	10,000	52,485	15,297	-0.02	47,195
40%	5,413	4,763	88%	10%	10,000	54,413	20,224	-0.01	45,614
	Self-protection								
$\frac{\sigma(\widetilde{\varepsilon})}{w_2}$	x^*	x^π	x^π/x^*	$p(x^*)$	L_0	$\mathbb{E}\widetilde{c}_2$	$\sigma(\widetilde{c}_2)$	$sk(\widetilde{c}_2)$	c_{sce}
0%	417	0	0%	5.27%	10,000	49,473	2,235	-4.00	49,466
10%	447	30	7%	5.03%	10,000	49,497	5,457	-0.26	49,207
20%	538	122	23%	4.38%	10,000	49,562	10,207	-0.04	48,408
30%	695	279	40%	3.44%	10,000	49,656	15,110	-0.01	47,005
40%	927	511	55%	2.41%	10,000	49,759	20,059	-0.00	44,876
	Self-insurance								
$\frac{\sigma(\widetilde{\varepsilon})}{w_2}$	y^*	y^π	y^π/y^*	p_0	$L(y^*)$	$\mathbb{E}\widetilde{c}_2$	$\sigma(\widetilde{c}_2)$	$sk(\widetilde{c}_2)$	c_{sce}
0%	389	0	0%	10%	6,061	49,394	1,218	-2.67	49,465
10%	418	29	7%	10%	5,843	49,416	5,298	-0.10	49,206
20%	503	114	23%	10%	5,234	49,477	10,123	-0.01	48,409
30%	647	258	40%	10%	4,351	49,565	15,057	-0.00	47,010
40%	853	464	54%	10%	3,337	49,666	20,025	-0.00	44,888

Table 4: Precautionary saving, self-protection and self-insurance in the base case with symmetric income risks $\widetilde{\varepsilon} = [0.5, -\varepsilon; 0.5, \varepsilon]$. The ε values of \$0, \$5,000, \$10,000, \$15,000 and \$20,000 yield a 0%, 10%, 20%, 30% and 40% standard deviation of second-period income.

The instruments affect the distribution of second-period consumption in different ways as summarized in Remark 1. Saving increases expected consumption but has no effect on the standard deviation and the skewness of second-period consumption; self-protection and self-insurance have the added benefit of reducing the standard deviation and increasing the skewness of second-period consumption.[16] This explains why the DM uses saving more than self-protection or self-insurance, namely to compensate for the fact that saving does not mitigate the riskiness or downside riskiness of second-period consumption.

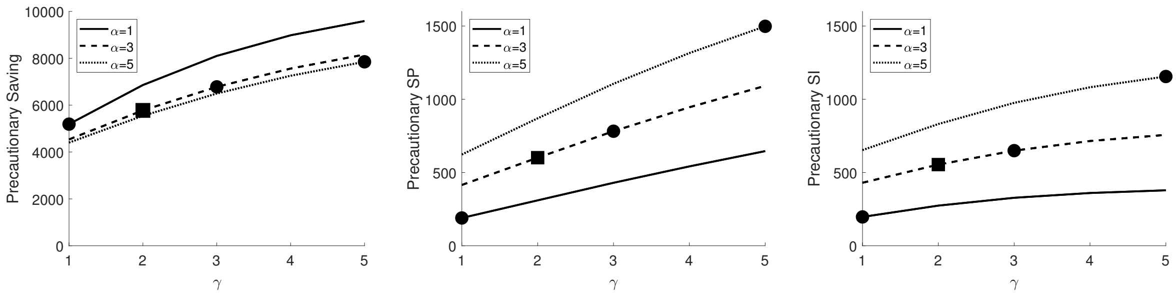
[16] The only exception is self-protection in the absence of income risk. Starting from $\mathrm{d}sk(\widetilde{c}_2)/\mathrm{d}x$ and $\mathrm{d}sk(\widetilde{c}_2)/\mathrm{d}y$ in Online Appendix C.1, self-protection lowers $sk(\widetilde{c}_2)$ in the absence of income risk and self-insurance has no effect on $sk(\widetilde{c}_2)$ in the absence of income risk.

	Saving			Loss risk		Moments of $\widetilde{c}_2$			
$sk(\widetilde{\varepsilon})$	s^*	s^π	s^π/s^*	p_0	L_0	$\mathbb{E}\widetilde{c}_2$	$\sigma(\widetilde{c}_2)$	$sk(\widetilde{c}_2)$	c_{sce}
0	2,669	2,018	76%	10%	10,000	51,668	12,855	-0.03	47,845
-0.5	4,420	3,769	85%	10%	10,000	53,420	12,855	-0.49	46,133
-1.0	6,427	5,776	90%	10%	10,000	55,427	12,855	-0.95	44,217
-1.5	8,692	8,042	93%	10%	10,000	57,692	12,855	-1.41	42,085
-2.0	11,211	10,560	94%	10%	10,000	60,211	12,855	-1.87	39,731
	Self-protection								
$sk(\widetilde{\varepsilon})$	x^*	x^π	x^π/x^*	$p(x^*)$	L_0	$\mathbb{E}\widetilde{c}_2$	$\sigma(\widetilde{c}_2)$	$sk(\widetilde{c}_2)$	c_{sce}
0	608	191	31%	3.93%	10,000	49,607	12,650	-0.02	47,788
-0.5	784	367	47%	3.00%	10,000	49,700	12,616	-0.50	45,733
-1.0	1,018	601	59%	2.09%	10,000	49,791	12,582	-0.99	43,118
-1.5	1,342	925	69%	1.27%	10,000	49,873	12,550	-1.49	39,712
-2.0	1,821	1,404	77%	0.61%	10,000	49,939	12,524	-1.99	35,159
	Self-insurance								
$sk(\widetilde{\varepsilon})$	y^*	y^π	y^π/y^*	p_0	$L(y^*)$	$\mathbb{E}\widetilde{c}_2$	$\sigma(\widetilde{c}_2)$	$sk(\widetilde{c}_2)$	c_{sce}
0	568	178	31%	10%	4,818	49,518	12,593	-0.00	47,791
-0.5	730	341	47%	10%	3,908	49,609	12,555	-0.50	45,737
-1.0	943	554	59%	10%	2,971	49,703	12,532	-0.99	43,125
-1.5	1,230	840	68%	10%	2,056	49,794	12,515	-1.49	39,725
-2.0	1,628	1,239	76%	10%	1,231	49,877	12,505	-2.00	35,182

Table 5: Precautionary saving, self-protection and self-insurance in the base case with skewed income risks $\widetilde{\varepsilon} = [q, \varepsilon_-; (1-q), \varepsilon_+]$ with $\mathbb{E}\widetilde{\varepsilon} = 0$ and $\sigma(\widetilde{\varepsilon})/w_2 = 25\%$. Parameters $q \in (0,1)$, ε_- and ε_+ are uniquely determined by Ebert's (2015) Proposition 1 to obtain skewness values ranging from 0 to -2, see Table 9 in Online Appendix C.2.

In terms of welfare, income risk reduces smooth certainty-equivalent consumption at an increasing rate for all three instruments. At low income risk levels ($\leq 10\%$), self-protection and self-insurance are more valuable for the DM than saving, but as income risk increases, this pattern reverses. Where this reversal occurs depends on the efficiency parameters μ and ν. On average, increasing the standard deviation of the income risk by 1 dollar reduces c_{sce} by 19 cents for saving and by 22 cents for self-protection and self-insurance.

Table 5 shows the impact of downside risk on precautionary behavior. As Proposition 3 predicts, saving, self-protection and self-insurance increase in the downside riskiness of the income risk. The precautionary components increase with downside risk at an increasing rate.



(a) Saving (b) Self-protection (c) Self-insurance

Figure 1: Precautionary choices in the single-instrument cases for various values of α and γ. The underlying income risk is skewed with $\mathbb{E}\widetilde{\varepsilon} = 0$, $\sigma(\widetilde{\varepsilon}) = \$12,500$, and $sk(\widetilde{\varepsilon}) = -1$. The square represents the values of s^{π}, x^{π} and y^{π} from the base case ($\alpha = 3, \gamma = 2$), see the third row in Table 5. The dots represent additively separable expected utility with $\alpha = \gamma$.

For a skewness of -1, precautionary responses are stronger than in Table 4, and substantially so for high negative skewness. The precautionary response of saving exceeds that of self-protection and self-insurance by a factor of 8 to 11.

Self-protection and self-insurance now reduce the skewness of second-period consumption slightly, contrary to the case with symmetric income risks where they tend to increase it. According to Remark 1, the effect of self-protection on the skewness of second-period consumption depends on a probability threshold that becomes smaller as the skewness of the income risk decreases. In our example, this threshold is less than 1% for $sk(\widetilde{\varepsilon}) \leq -0.29$ and negative as soon as $sk(\widetilde{\varepsilon}) \leq -0.54$. The probability threshold for self-insurance is also decreasing in the downside riskiness of income risk. It is less than 1% for $sk(\widetilde{\varepsilon}) \leq -0.28$ and negative as soon as $sk(\widetilde{\varepsilon}) \leq -0.34$. Self-protection and self-insurance still have the added benefit of reducing the standard deviation of second-period consumption.

In terms of welfare, downside risk reduces smooth certainty-equivalent consumption at an increasing rate. Self-protection and self-insurance are almost equally effective at addressing the negative skewness of the income risk, yet saving is more effective. For high negative skewness, say $sk(\widetilde{\varepsilon}) = -2$, smooth certainty-equivalent consumption is 13% higher when the DM uses saving instead of self-protection or self-insurance. For saving, a one percentage point decrease in the skewness of the income risk has, on average, the same effect on the DM as a certain loss of \$40.57 in each period. This loss is \$63.15 for self-protection and \$63.05 for self-insurance, which is about one-and-a-half times as high as for saving. The advantage of saving over self-protection and self-insurance increases in the downside riskiness of the income risk. In our setting, welfare losses due to negatively skewed income risks can be substantially larger than welfare losses due to symmetric income risks.

In Figure 1, we show how precautionary saving, self-protection and self-insurance depend on the values of the preference parameters α and γ in the single-instrument cases. The underlying income risk is skewed with $sk(\widetilde{\varepsilon}) = -1$, corresponding to the third rows in Table 5. We choose this risk for illustration because the standard deviation and skewness of the associated log earnings growth fit particularly well within the ranges reported by De Nardi et al. (2020). Precautionary saving ranges from \$4,402 to \$9,589, precautionary self-protection from \$190 to \$1,498, and precautionary self-insurance from \$197 to \$1,156. Saving is more sensitive to changes in preference parameters, followed by self-protection and then self-insurance. All precautionary responses in Figure 1 are positive even when the CE channel is negative, that is, when $\alpha < \gamma$. The positive MEU channel always dominates in our setting. Furthermore, the amount of precaution is increasing in the relative risk aversion parameter γ for all three instruments. For iso-elastic utility, $\gamma + 1$ measures the degree of convexity of the marginal utility function, so the positive effect comes from the MEU channel.

The effect of EIS, namely $1/\alpha$, differs across instruments. Higher EIS increases precautionary saving, but decreases precautionary self-protection and precautionary self-insurance. The reason is related to the CE channel. If the CE of second-period consumption exceeds first-period consumption, utility is higher in the second period than in the first period. An increase in EIS reduces the curvature of the felicity function so that the marginal value of additional consumption is higher in the second period than in the first period. Table 5 tells us that DMs use saving more than self-protection and self-insurance. Higher instrument use implies lower first-period consumption and a higher CE in the second period, which explains why the effect of EIS differs across instruments (see also Huber, 2021, on this point).

The utility discount factor has no major impact on precautionary behavior. We varied β from 0.95 to 1 for the skewed income risk with $sk(\widetilde{\varepsilon}) = -1$. Instrument use is increasing in β for all instruments both in the absence and in the presence of income risk. More patient DMs are willing to spend more money upfront to increase expected consumption in the second period. For saving, the effect is slightly stronger in the absence of income risk so that precautionary saving is decreasing in β. However, the size of the effect is small and less than \$70. Precautionary self-protection and precautionary self-insurance are increasing in β but the effect is so small that it is hardly perceptible. Appendix B discusses the effect of return parameters R, μ and ν on precautionary behavior.

5.3 Precaution with two instruments

We now let two instruments be available to the DM. First, consider saving and self-protection. Let $(s^0, x^0) = \arg\max_{(s,x)} U(s, x; 0)$ be the optimal levels of saving and self-protection in the absence of income risk and $(s^*, x^*) = \arg\max_{(s,x)} U(s, x; \widetilde{\varepsilon})$ be the optimal levels of saving and self-protection in the presence of income risk. Precautionary saving and precautionary self-protection are then given by $s^\pi = s^* - s^0$ and $x^\pi = x^* - x^0$. We also report the total amount of precaution, $\pi_{s,x} = s^\pi + x^\pi$. To quantify interaction effects, we consider the restricted response of each instrument by keeping the other instrument fixed. This yields $s^r = \arg\max_s U(s, x^0; \widetilde{\varepsilon})$ for saving and $x^r = \arg\max_x U(s^0, x; \widetilde{\varepsilon})$ for self-protection, where superscript r is short for restricted. Restricted precautionary saving is $s^\pi_r = s^r - s^0$, and restricted precautionary self-protection is $x_r^\pi = x^r - x^0$. The comparison of s_r^π and s^π informs about the interaction effect of self-protection on saving, and the comparison of x_r^π and x^π about the interaction effect of saving on self-protection. These notations apply analogously to the other pairs of instruments.

	Saving			**Self-protection**				
$\frac{\sigma(\tilde{\varepsilon})}{w_2}$	s^*	s^π	s_r^π	x^*	x^π	x_r^π	$\pi_{s,x}$	c_{sce}
0%	154	0	0	405	0	0	0	49,467
10%	488	334	337	408	4	30	338	49,213
20%	1,448	1,294	1,308	418	13	121	1,307	48,471
30%	2,934	2,780	2,810	433	28	276	2,809	47,285
40%	4,827	4,673	4,724	452	47	507	4,720	45,713

	Saving			**Self-insurance**				
$\frac{\sigma(\tilde{\varepsilon})}{w_2}$	s^*	s^π	s_r^π	y^*	y^π	y_r^π	$\pi_{s,y}$	c_{sce}
0%	172	0	0	377	0	0	0	49,465
10%	504	332	335	380	4	28	336	49,213
20%	1,458	1,286	1,301	391	14	113	1,300	48,473
30%	2,935	2,763	2,794	406	29	255	2,792	47,291
40%	4,818	4,646	4,697	423	47	460	4,693	45,726

	Self-protection			**Self-insurance**				
$\frac{\sigma(\tilde{\varepsilon})}{w_2}$	x^*	x^π	x_r^π	y^*	y^π	y_r^π	$\pi_{x,y}$	c_{sce}
0%	250	0	0	163	0	0	0	49,468
10%	253	3	29	189	26	29	29	49,209
20%	268	18	117	261	98	113	115	48,413
30%	306	56	267	368	205	254	261	47,014
40%	382	132	488	502	338	455	470	44,893

Table 6: Precautionary behavior in the base case with two instruments for symmetric income risks $\tilde{\varepsilon} = [0.5, -\varepsilon; 0.5, \varepsilon]$. The ε values \$0, \$5,000, \$10,000 \$15,000 and \$20,000 yield a 0%, 10% , 20%, 30% and 40% standard deviation of second-period income.

Table 6 reports the results in the base case for symmetric income risks and Table 7 for downside risk. As in the single-instrument cases, precautionary saving, self-protection and self-insurance occur and increase in the riskiness and downside riskiness of the income risk. So the positive precautionary effect of income risk always dominates negative substitution effects of one instrument on the other. Saving exerts strong substitution effects on self-protection and self-insurance and can reduce their precautionary response by more than 90%, especially for skewed income risks. Self-protection and self-insurance, in contrast, exert only moderate

	Saving			Self-protection				
$sk(\widetilde{\varepsilon})$	s^*	s^π	s_r^π	x^*	x^π	x_r^π	$\pi_{s,x}$	c_{sce}
0	2,133	1,979	2,000	425	20	190	1,999	47,930
-0.5	3,870	3,717	3,747	433	28	365	3,745	46,222
-1.0	5,856	5,702	5,745	445	40	597	5,742	44,311
-1.5	8,092	7,938	7,999	461	56	919	7,994	42,187
-2.0	10,570	10,416	10,502	482	77	1,392	10,493	39,844

	Saving			Self-insurance				
$sk(\widetilde{\varepsilon})$	s^*	s^π	s_r^π	y^*	y^π	y_r^π	$\pi_{s,y}$	c_{sce}
0	2,139	1,966	1,988	398	21	177	1,987	47,934
-0.5	3,872	3,700	3,731	406	29	339	3,729	46,228
-1.0	5,852	5,680	5,723	417	40	550	5,720	44,321
-1.5	8,079	7,907	7,968	432	55	834	7,962	42,202
-2.0	10,546	10,374	10,457	451	74	1,228	10,448	39,867

	Self-protection			Self-insurance				
$sk(\widetilde{\varepsilon})$	x^*	x^π	x_r^π	y^*	y^π	y_r^π	$\pi_{x,y}$	c_{sce}
0	283	33	183	311	147	176	180	47,795
-0.5	372	122	354	388	225	333	347	45,743
-1.0	479	229	579	496	332	538	664	43,133
-1.5	619	369	886	641	478	817	847	39,735
-2.0	815	565	1,329	836	672	1,205	1,237	35,193

Table 7: Precautionary behavior in the base case with two instruments for skewed income risks $\widetilde{\varepsilon} = [q, \varepsilon_-; (1-q), \varepsilon_+]$ with $\mathbb{E}\widetilde{\varepsilon} = 0$ and $\sigma(\widetilde{\varepsilon})/w_2 = 25\%$. Parameters $q \in (0,1)$, ε_- and ε_+ are uniquely determined to obtain skewness values ranging from 0 to -2, see Table 9 in Online Appendix C.2.

substitution effects on saving, reducing the amount of precautionary saving by roughly 1% or less. For symmetric income risks, the substitution effect of self-insurance on self-protection is stronger than that of self-protection on self-insurance. As the negative skewness of the income risk increases, the two substitution effects become more equal in size. For example, for $sk(\widetilde{\varepsilon}) = -1$ (third rows in Table 7), self-insurance reduces precautionary self-protection by 60% from \$579 to \$229 and self-protection reduces precautionary self-insurance by 38% from \$538 to \$332. In our setting, self-protection is most susceptible to substitution effects, followed by self-insurance and then saving, which is quite robust to substitution effects.

The total precautionary response is increasing in the riskiness and downside riskiness of the income risk. For each pair of instruments, its level is higher than the precautionary response of the less sensitive instrument but lower than that of the more sensitive instrument in the single-instrument cases. For example, for saving and self-protection with $sk(\widetilde{\varepsilon}) = -1$, we have $s^{\pi} = \$5,776$ and $x^{\pi} = \$601$ from Table 5 in the single-instrument cases, and $\pi_{s,x} = \$5,742$ from Table 7 when both instruments are used simultaneously. We then observe that $\pi_{s,x}$ exceeds x^{π} but is less than s^{π}.

Smooth certainty-equivalent consumption is higher than in any of the corresponding single-instrument cases because the availability of an additional instrument can never make the DM worse off. The DM gains most from access to saving, and the additional value increases at an increasing rate in the riskiness and downside riskiness of the income risk. For symmetric income risks, saving increases c_{sce} up to \$838 and for skewed income risks the gain can be as high as \$4,685.[17] By contrast, the gains from access to self-protection or self-insurance on top of saving are more moderate, in the ranges of \$73-\$112 for symmetric income risks and \$85-\$136 for skewed income risks. Having access to self-insurance on top of self-protection or to self-protection on top of self-insurance yields even smaller welfare gains that do not exceed \$15 in most cases.

5.4 Precaution with three instruments

Now assume that all three instruments are available to the DM. In the absence of income risk, the optimal choice is $(s^0, x^0, y^0) = \arg\max_{(s,x,y)} U(s, x, y; 0)$, in the presence of income risk, it is $(s^*, x^*, y^*) = \arg\max_{(s,x,y)} U(s, x, y; \widetilde{\varepsilon})$. Precautionary saving, self-protection and self-insurance are given by $s^{\pi} = s^* - s^0$, $x^{\pi} = x^* - x^0$ and $y^{\pi} = y^* - y^0$, respectively, and the total amount of precaution is $\pi_{s,x,y} = s^{\pi} + x^{\pi} + y^{\pi}$. The restricted responses are now $s^r = \arg\max_s U\left(s, x^0, y^0; \widetilde{\varepsilon}\right)$ for saving, $x^r = \arg\max_x U\left(s^0, x, y^0; \widetilde{\varepsilon}\right)$ for self-protection, and $y^r = \arg\max_y U\left(s^0, x^0, y; \widetilde{\varepsilon}\right)$ for self-insurance. For the restricted responses, we keep both of the other instruments at their level without income risk to isolate the direct effect of income risk on the instrument under consideration. The restricted precautionary choices are $s^{\pi}_r = s^r - s^0$ for saving, $x^{\pi}_r = x^r - x^0$ for self-protection, and $y^{\pi}_r = y^r - y^0$ for self-insurance. Comparing s^{π}_r and s^{π} informs us about the joint substitution effect of self-protection and self-insurance on saving, and likewise for the other instruments.

Table 8 presents the results in the base case for symmetric and skewed income risks when all three instruments are available to the DM. Positive precautionary reactions now only arise in saving and self-insurance, and both are increasing in the riskiness and downside riskiness of the income risk. As in the two-instrument cases, the substitution effects of self-protection and self-insurance on saving are hardly perceptible and reduce precautionary saving

[17] When $\sigma(\widetilde{\varepsilon})/w_2 = 40\%$, we obtain \$837 by subtracting \$44,876 in Table 4 for self-protection from \$45,713 in Table 6 for saving and self-protection, and \$838 by subtracting \$44,888 in Table 4 for self-insurance from \$45,726 in Table 6 for saving and self-insurance. For skewness with $sk(\widetilde{\varepsilon}) = -2$, we find \$4,685 = \$39,844-\$35,159 for being able to use saving in addition to self-protection, and the same \$4,685 = \$39,867-\$35,182 for being able to use saving in addition to self-insurance, see Tables 5 and 7.

	Saving			Self-protection			Self-insurance				
$\frac{\sigma(\tilde{\varepsilon})}{w_2}$	s^*	s^π	s_r^π	x^*	x^π	x_r^π	y^*	y^π	y_r^π	$\pi_{s,x,y}$	c_{sce}
0%	149	0	0	241	0	0	161	0	0	0	49,468
10%	482	333	336	223	-18	28	181	20	28	335	49,216
20%	1,440	1,291	1,304	177	-64	116	234	53	112	1,280	48,475
30%	2,923	2,774	2,801	116	-125	265	304	143	252	2,792	47,292
40%	4,813	4,664	4,709	50	-191	484	380	219	452	4,692	45,726

	Saving			Self-protection			Self-insurance				
$sk(\tilde{\varepsilon})$	s^*	s^π	s_r^π	x^*	x^π	x_r^π	y^*	y^π	y_r^π	$\pi_{s,x,y}$	c_{sce}
0	2,124	1,975	1,994	147	-94	182	268	107	175	1,988	47,935
-0.5	3,860	3,711	3,738	116	-125	352	304	143	331	3,729	46,229
-1.0	5,843	5,694	5,733	74	-167	575	353	192	536	5,719	44,321
-1.5	8,077	7,928	7,981	20	-221	880	414	253	812	7,960	42,202
-2.0	10,547	10,398	10,476	0	-241	1,320	450	289	1,197	10,446	39,867

Table 8: Precautionary behavior in the base case with all three instruments. The first panel is for symmetric income risks $\tilde{\varepsilon} = [0.5, -\varepsilon;\, 0.5, \varepsilon]$. The ε values \$0, \$5,000, \$10,000 \$15,000 and \$20,000 yield a 0%, 10% , 20%, 30% and 40% standard deviation of second-period income. The second panel is for skewed income risks $\tilde{\varepsilon} = [q, \varepsilon_-;\, (1-q), \varepsilon_+]$ with $\mathbb{E}\tilde{\varepsilon} = 0$ and $\sigma(\tilde{\varepsilon})/w_2 = 25\%$. Parameters $q \in (0,1)$, ε_- and ε_+ generate skewness values ranging from 0 to -2, see Table 9 in Online Appendix C.2.

by less than 1%. The substitution effect of saving and self-protection on self-insurance lowers precautionary self-insurance by 29-52% for symmetric income risks and by 39-76% for skewed income risks. It is sizeable but smaller than the substitution effect of saving on self-insurance in the corresponding two-instrument case. The substitution effect of saving and self-insurance on self-protection is so strong that it outweighs the positive precautionary effect of income risk. Optimal self-protection is now decreasing in the riskiness and downside riskiness of the income risk. This results in negative values for x^π, despite the fact that the conditions in Proposition 3(i) are satisfied. For income risks with high negative skewness, saving and self-insurance crowd out self-protection entirely. This example shows that the DM's instrument portfolio has a major impact on the link between preferences and precautionary behavior. The role of substitution effects ranges from hardly perceptible to being of first-order importance in the sense that they can dominate precautionary effects. The case of self-protection illustrates that substitution effects can turn existing predictions upside down.

The total precautionary response is increasing in the riskiness and downside riskiness of the income risk. Its level is higher than the least sensitive pairing of instruments, but lower than the two more sensitive pairings of instruments from the two-instrument cases. Specifically, in our set-up, $\pi_{s,x,y}$ in Table 8 exceeds the corresponding $\pi_{x,y}$-value but is less than the corresponding value for $\pi_{s,x}$ and $\pi_{s,y}$ in Tables 6 and 7. For example, when $sk(\widetilde{\varepsilon}) = -1$, we have $\pi_{s,x,y} = \$5,719$ in Table 8, which is larger than $\pi_{x,y} = \$664$ in Table 7 but smaller than $\pi_{s,x} = \$5,742$ and $\pi_{s,y} = \$5,720$ in the same table.

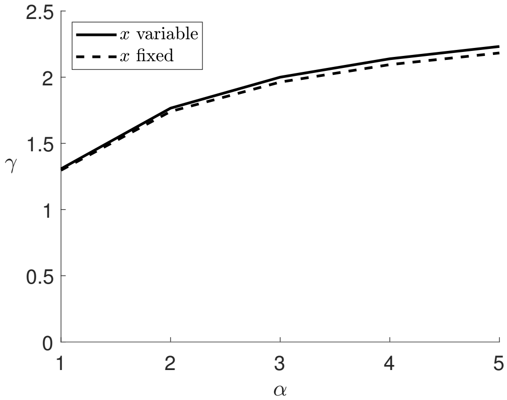
In terms of welfare, the DM benefits from being able to use all three instruments instead of only two, yet the gains differ across instruments. Access to saving is most valuable but the gains are slightly smaller than in the two-instrument case. For symmetric income risks, saving increases c_{sce} up to \$833 and for skewed income risks the gain is up to \$4,674. The gain from access to self-insurance does not exceed \$15 in all but one case, and access to self-protection raises c_{sce} even less due to strong substitution effects. When saving and self-insurance crowd out self-protection to a large extent anyway, access to self-protection is only of little value.

6 Inference

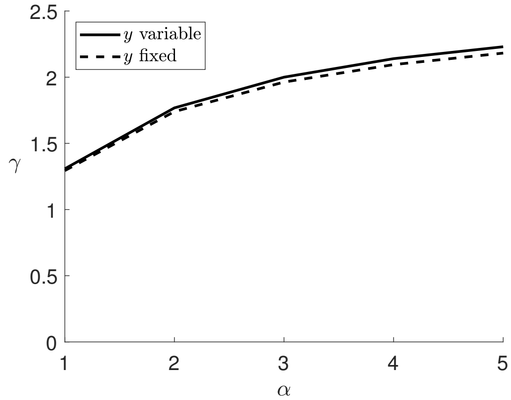
In this section, we provide further insight into the link between preferences and precautionary behavior and the role of instrument interaction in this relationship. Section 5 focused on the effects of riskiness and downside riskiness on instrument use for a given set of RU preferences. In this section, we wonder about different RU preferences that can explain a precautionary motive of a given size when either one or two instruments are available to the DM.

We focus on the zero-mean income risk with $\sigma(\widetilde{\varepsilon})/w_2 = 25\%$ and $sk(\widetilde{\varepsilon}) = -1$ because the standard deviation and skewness of the associated log earnings growth fit nicely in the empirical ranges given by De Nardi et al. (2020). This income risk was used in the third rows of Tables 5 and 7, and in the third row of the bottom panel of Table 8. We fix the precautionary use of an instrument and identify the (α, γ)-combinations that lead to the same amount of precaution in that instrument. Hence, we determine iso-precaution curves in the (α, γ)-plane. If an additional instrument is available to the DM, these iso-curves can be determined in at least two different ways. We can let the other instrument adjust endogenously as we vary the preference parameters. Alternatively, we can keep the other instrument fixed at its baseline level so that the first instrument absorbs the entire precautionary response. We use Kimball and Weil's (KW, 2009) relative prudence measure for RU to assess the discrepancy between the two curves. This measure is given by $\gamma\,(1+\alpha)\,/\alpha$, see their Section 2.

Figure 2 focuses on precautionary saving and considers the presence of either self-protection in panel (a) or self-insurance in panel (b). For saving and self-protection, we have $s^* = \$5,856$, $x^* = \$445$ and precautionary savings of $s^{\pi} = \$5,702$ from the top panel of Table 7 for RU preferences with $\alpha = 3$ and $\gamma = 2$. The solid curve collects the (α, γ)-combinations that lead to the same amount of precautionary saving while letting the level of self-protection vary as we adjust preferences. The dashed curve collects the (α, γ)-combinations that lead to the same amount of precautionary saving but keeping self-protection fixed at its level in the base case, $x^* = \$445$. The dashed curve lies slightly below the solid curve resulting in smaller values of relative KW-prudence. If saving absorbs the entire precautionary response, less prudence suffices to generate the given amount of precautionary saving. If the other instrument is en-



(a) Iso-precautionary saving curves with endogenous self-protection (solid line) or a fixed level of self-protection $x = \$445$ (dashed line)



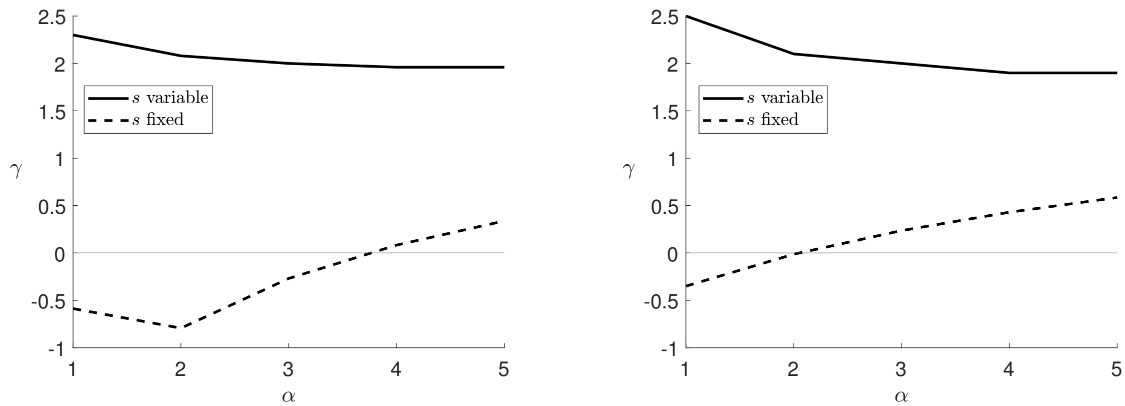
(b) Iso-precautionary saving curves with endogenous self-insurance (solid line) or a fixed level of self-insurance $y = \$417$ (dashed line)

Figure 2: Iso-precautionary saving curves in the (α, γ)-plane. The solid curve allows for adjustments to the other instrument, the dashed curve keeps the other instrument fixed.

dogenous, a substitution effect is at work, which diminishes precautionary saving. Therefore, more prudence is necessary to obtain the same precautionary saving amount. The difference is small in magnitude though. Along the solid line, relative KW-prudence varies from 2.62 to 2.68, and along the dashed line, it varies from 2.59 to 2.62.

Matters are similar in panel (b) for precautionary saving and self-insurance. We have $s^* = \$5,852$, $y^* = \$417$ and precautionary savings of $s^\pi = \$5,680$ from the middle panel of Table 7. The two curves in panel (b) collect the (α, γ)-combinations that lead to the same precautionary saving amount. The solid curve allows self-insurance to adjust as preference parameters vary; the dashed curve keeps self-insurance fixed at its level in the base case. The solid curve lies above the dashed curve due to the substitution effect when self-insurance is endogenous. Relative KW-prudence ranges from 2.59 to 2.62 along the solid curve and from 2.62 to 2.68 along the dashed curve. The γ-values in panel (b) are higher than those in panel (a) but the difference is hardly perceptible and so small that relative KW-prudence is identical up to the first two decimal places. Even though we see a difference between the two curves in panels (a) and (b), saving is fairly robust to substitution effects from self-protection and self-insurance, and instrument interaction only has subtle effects on preference identification.

Figure 3 considers the reverse scenarios of precautionary self-protection and precautionary self-insurance in the presence of saving. For self-protection and saving, we have $x^* = \$445$, $s^* = \$5,856$ and precautionary self-protection of $x^\pi = \$40$ from the top panel of Table 7. The two curves in panel (a) collect the (α, γ)-combinations where x^π remains unchanged, the solid curve allows saving to vary, the dashed curve keeps saving fixed at its level in the base case. Now we see a pronounced gap between the two curves. Relative KW-prudence ranges from 4.6 to 2.17 along the solid curve and from -1.19 to 0.92 along the dashed curve. The substitution effect of saving on self-protection is so strong that the precautionary response of



(a) Iso-precautionary self-protection curves with endogenous saving (solid line) or a fixed level of saving $s = \$5,856$ (dashed line)

(b) Iso-precautionary self-insurance curves with endogenous saving (solid line) or a fixed level of saving $s = \$5,852$ (dashed line)

Figure 3: Iso-precautionary self-protection and self-insurance curves in the (α, γ)-plane. The solid curve allows for adjustments to saving, the dashed curve keeps saving fixed.

self-protection looks small. Trying to explain such a modest precautionary response without integrating other instruments requires much lower levels of prudence. In panel (a), the dashed curve even requires negative γ-values for $\alpha \leq 3.5$ because positive values of γ cannot generate amounts of precautionary self-protection that low.

For self-insurance and saving, we have $y^* = \$417$, $s^* = \$5,852$ and precautionary self-insurance of $y^\pi = \$40$ from the middle panel of Table 7. The two curves in Panel (b) collect the (α, γ)-combinations where y^π remains unchanged, the solid curve allows saving to vary, the dashed curve keeps saving fixed at its level in the base case. Relative KW-prudence ranges from 5 to 2.12 along the solid curve and from -0.70 to 1.03 along the dashed curve. Saving exerts a strong substitution effect on self-insurance, which reduces its precautionary response significantly and lowers the implied levels of prudence substantially. For $\alpha \leq 2.5$, the dashed curve even requires negative γ-values because positive values for γ would generate higher amounts of precautionary self-insurance.

These results caution the wary empiricist. Of course, when the DM's instrument portfolio is perfectly known, none of this matters because one can simply model the link between preferences and behavior. If one is uncertain about which instruments DMs use to react to income risk, or if instrument portfolios differ across DMs, spurious conclusions about the magnitude of underlying precautionary preferences may be the result. In the context of saving, self-protection and self-insurance, our results show that some instruments are more likely than others to be subject to strong portfolio effects.

7 Conclusion

We analyze precautionary behavior in a model that disentangles risk and time. DMs can use various instruments to deal with income risk: saving, self-protection and self-insurance. We derive a unifying result and show that, when used in isolation, all three instruments are subject to the same trade-offs as the level of income risk changes. Our result encompasses higher-order risk effects and we study risk averters and risk lovers alike. When instruments are used in combination, substitutive interaction effects arise that impede general conclusions. We thus provide a detailed numerical analysis to explore and compare precautionary behavior across instruments and evaluate how instruments interact.

In our setting, saving shows the largest precautionary response and is quite robust to substitution effects. It is well-suited to infer preferences from precautionary motives even when researchers are unsure whether and how DMs incorporate self-protection and self-insurance into their overall life-cycle optimization. Matters are different for precautionary self-protection and self-insurance. Both instruments show a more moderate precautionary response in isolation and experience strong substitution effects from saving. The substitution effect can be strong enough to outweigh precautionary effects and lower instrument use, even when the underlying preferences ensure precautionary behavior in single-instrument scenarios. This susceptibility to substitution effects makes self-protection and self-insurance less suited to identify underlying preferences. In our setting, substitution effects can lead to levels of precautionary self-protection or self-insurance that are so low that empiricists may suspect negative values of relative prudence if it were not for other instruments.

More generally, our paper highlights the need to think carefully about a DM's portfolio of instruments. The set of instruments can have important implications for the prediction of precautionary behavior and the inference of preferences from precautionary choices. People engage in different kinds of behaviors when they anticipate and respond to income risk. The portfolio of instruments may also differ across individuals, with some people responding more broadly to income risk than others. Even when instruments are subject to the same qualitative trade-offs, they may differ to a large extent in their interaction. While challenging from an empirical standpoint, we are confident that the deliberate consideration of instrument portfolios will help improve our understanding of precautionary motives in the future.

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A Proofs

A.1 Proof of Proposition 1

We show that condition (i) implies that $U(s;\widetilde{\varepsilon})$ dominates $U(s;0)$ on $[-(w_2+\underline{\varepsilon}-L)/R, w_1]$ by the interval dominance order while condition (ii) implies the reverse ordering.[18] Take s'' and s' with $s''>s'$ and $U(s'';0)\geq U(s;0)$ for all $s\in[s',s'']$. Then

$$U(s'';0)-U(s';0)\geq 0 \qquad \Longrightarrow \qquad U(s'';\widetilde{\varepsilon})-U(s';\widetilde{\varepsilon})\geq 0$$

if, equivalently,

$$\begin{aligned} & \beta u\left(CE(w_2+s''R+\widetilde{\ell})\right)-\beta u\left(CE(w_2+s'R+\widetilde{\ell})\right) && \geq && u(w_1-s')-u(w_1-s'') \\ \Longrightarrow \quad & \beta u\left(CE(\widetilde{w}_2+s''R+\widetilde{\ell})\right)-\beta u\left(CE(\widetilde{w}_2+s'R+\widetilde{\ell})\right) && \geq && u(w_1-s')-u(w_1-s''). \end{aligned}$$

Sufficient for the last implication is that

$$\begin{aligned} & u\left(CE(\widetilde{w}_2+s''R+\widetilde{\ell})\right)-u\left(CE(\widetilde{w}_2+s'R+\widetilde{\ell})\right) \\ \geq \; & u\left(CE(w_2+s''R+\widetilde{\ell})\right)-u\left(CE(w_2+s'R+\widetilde{\ell})\right). \end{aligned} \tag{3}$$

Inequality (3) is satisfied if, for all $s\in[s',s'']$,

$$u'\left(CE(\widetilde{w}_2+sR+\widetilde{\ell})\right)\frac{\mathrm{d}CE(\widetilde{w}_2+sR+\widetilde{\ell})}{\mathrm{d}s}\geq u'\left(CE(w_2+sR+\widetilde{\ell})\right)\frac{\mathrm{d}CE(w_2+sR+\widetilde{\ell})}{\mathrm{d}s}.$$

With the help of the implicit function rule, we can rewrite this as follows:

$$\frac{u'\left(CE(\widetilde{w}_2+sR+\widetilde{\ell})\right)}{\psi'\left(CE(\widetilde{w}_2+sR+\widetilde{\ell})\right)}\mathbb{E}\psi'(\widetilde{w}_2+sR+\widetilde{\ell})\geq\frac{u'\left(CE(w_2+sR+\widetilde{\ell})\right)}{\psi'\left(CE(w_2+sR+\widetilde{\ell})\right)}\mathbb{E}\psi'(w_2+sR+\widetilde{\ell}).$$

Now $\widetilde{w}_2+sR+\widetilde{\ell}$ is riskier than $w_2+sR+\widetilde{\ell}$ in the sense of Rothschild and Stiglitz (1970), and the concavity of ψ implies a lower CE for $\widetilde{w}_2+sR+\widetilde{\ell}$ than for $w_2+sR+\widetilde{\ell}$. This decrease in CE raises the ratio of marginal utilities if u is more concave than ψ. Finally, convexity of ψ' ensures that expected marginal utility is higher for the riskier consumption distribution, which completes the proof of (i).

To demonstrate (ii), the same reasoning as above shows that the reverse of condition (3) is sufficient for $U(s;0)$ to dominate $U(s;\widetilde{\varepsilon})$ on $[-(w_2+\underline{\varepsilon}-L)/R, w_1]$ by the interval dominance order. The lower certainty equivalent associated with $\widetilde{w}_2+sR+\widetilde{\ell}$ lowers the ratio of marginal

[18] The domain of $U(s;\widetilde{\varepsilon})$ is smaller than the domain of $U(s;0)$ because it does not contain values between $-(w_2-L)/R$ and $-(w_2+\underline{\varepsilon}-L)/R$. This does not affect result (i) but may affect result (ii) if $U(s;0)$ has maximizers smaller than $-(w_2+\underline{\varepsilon}-L)/R$. In this case, (ii) holds on the intersection of both domains.

utilities if u is less concave than ψ. Concavity of ψ' results in lower expected marginal utility for the riskier consumption distribution. Combining the two effects completes the proof.

A.2 Proof of Remark 2

If ψ has CARA, we can write $\psi(c_2) = 1 - \exp(-\alpha c_2)$ for $\alpha > 0$. Due to the independence of $\widetilde{\varepsilon}$ and $\widetilde{\ell}$, we then obtain

$$CE(\widetilde{w}_2 + s'R + \widetilde{\ell}) = w_2 - \frac{1}{\alpha}\ln \mathbb{E}\exp(-\alpha\widetilde{\varepsilon}) + s'R - \frac{1}{\alpha}\ln \mathbb{E}\exp(-\alpha\widetilde{\ell}),$$

and likewise for s'' instead of s'. Inspecting inequality (3), we see that on both sides CE increases by $(s'' - s')R$. Income risk reduces CE because $\ln \mathbb{E}\exp(-\alpha\widetilde{\varepsilon}) > 0$, so on the left-hand side of (3) the increase by $(s'' - s')R$ occurs at a lower level than on the right-hand-side of (3). It then follows from the concavity of u that the utility gap is larger on the left-hand side than on the right-hand side so that the inequality is indeed satisfied.

For self-protection and self-insurance, the argument is similar. An increase in either activity lowers $\ln \mathbb{E}\exp(-\alpha\widetilde{\ell})$, which raises the certainty equivalent. This increase in CE raises felicity by more when CE is low rather than high. Raising CE is thus more valuable in the presence of income risk than in its absence, which explains the precautionary demand for self-protection and self-insurance when income risk is introduced.

A.3 Proof of Proposition 2

In the case of self-protection, the same steps as in Appendix A.1 show that a sufficient condition for $U(x;\widetilde{\varepsilon})$ to dominate $U(x;0)$ on $[0, w_1]$ by the interval dominance order is

$$u'\left(CE(\widetilde{w}_2 + \widetilde{\ell})\right)\frac{\mathrm{d}CE(\widetilde{w}_2 + \widetilde{\ell})}{\mathrm{d}x} \geq u'\left(CE(w_2 + \widetilde{\ell})\right)\frac{\mathrm{d}CE(w_2 + \widetilde{\ell})}{\mathrm{d}x}$$

for all $x \in [x', x'']$ with $x'' > x'$. We rewrite this with the help of the implicit function rule as:

$$\begin{aligned} & \frac{u'\left(CE(\widetilde{w}_2 + \widetilde{\ell})\right)}{\psi'\left(CE(\widetilde{w}_2 + \widetilde{\ell})\right)}\left(-p'(x)\left[\mathbb{E}\psi(\widetilde{w}_2) - \mathbb{E}\psi(\widetilde{w}_2 - L)\right]\right) \\ \geq \quad & \frac{u'\left(CE(w_2 + \widetilde{\ell})\right)}{\psi'\left(CE(w_2 + \widetilde{\ell})\right)}\left(-p'(x)\left[\psi(w_2) - \psi(w_2 - L)\right]\right). \end{aligned}$$

Concavity of ψ implies that the CE of $\widetilde{w}_2 + \widetilde{\ell}$ is lower than the CE of $w_2 + \widetilde{\ell}$. If u is more concave than ψ, this decrease in CE raises the ratio of marginal utilities. Furthermore, if ψ' is convex, income risk raises the utility difference between the no-loss state and the loss state. Combining both effects yields (i).

For (ii), the lower CE associated with $\widetilde{w}_2 + \widetilde{\ell}$ now lowers the ratio of marginal utilities because u is assumed to be less concave than ψ. Moreover, concavity of ψ' implies that income risk lowers the utility difference between the no-loss state and the loss state. This reverses the above inequality and implies that $U(x;0) \succeq_I U(w;\widetilde{\varepsilon})$ on $[0, w_1]$.

In case of self-insurance, similar arguments show that condition (i) implies

$$\frac{u'\left(CE(\widetilde{w}_2+\widetilde{\ell})\right)}{\psi'\left(CE(\widetilde{w}_2+\widetilde{\ell})\right)}\left(-L'(y)\mathbb{E}\psi'(\widetilde{w}_2-L(y))\right) \geq \frac{u'\left(CE(w_2+\widetilde{\ell})\right)}{\psi'\left(CE(w_2+\widetilde{\ell})\right)}\left(-L'(y)\psi'(w_2-L(y))\right),$$

while condition (ii) yields the reverse inequality for any $y \in [0, w_1]$. So $U(y;\widetilde{\varepsilon}) \succeq_I U(y;0)$ holds under (i) whereas $U(y;0) \succeq_I U(y;\widetilde{\varepsilon})$ holds under (ii). Theorem 1 completes the proof.

A.4 Proof of Proposition 3

We rank $U(a;\widetilde{\varepsilon}')$ and $U(a;\widetilde{\varepsilon}'')$ on $[\underline{a}, \overline{a}]$ by the interval dominance order. Take a'' and a' with $a'' > a'$ and $U(a'';\widetilde{\varepsilon}') \geq U(a;\widetilde{\varepsilon}')$ for all $a \in [a', a'']$. Then

$$U(a'';\widetilde{\varepsilon}') - U(a';\widetilde{\varepsilon}') \geq 0 \quad \Longrightarrow \quad U(a'';\widetilde{\varepsilon}'') - U(s';\widetilde{\varepsilon}'') \geq 0$$

if, equivalently,

$$\begin{aligned} \beta u\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_{a''})\right) - \beta u\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_{a'})\right) &\geq u(w_1-a') - u(w_1-a'') \\ \Longrightarrow \quad \beta u\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_{a''})\right) - \beta u\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_{a'})\right) &\geq u(w_1-a') - u(w_1-a''), \end{aligned}$$

where $\widetilde{\ell}_{a'}$ and $\widetilde{\ell}_{a''}$ are distributed according to $F(\ell;a')$ and $F(\ell;a'')$. This implication holds if

$$\begin{aligned} & u\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_{a''})\right) - u\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_{a'})\right) \\ \geq\ & u\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_{a''})\right) - u\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_{a'})\right). \end{aligned} \tag{4}$$

We introduce $H(\ell;t) = tF(\ell;a'') + (1-t)F(\ell;a')$ for $t \in [0,1]$ to parameterize the change from $F(\ell;a')$ to $F(\ell;a'')$, see Jindapon and Neilson (2007). Let $\widetilde{\ell}_t$ be distributed according to $H(\ell;t)$ and use the fundamental theorem of calculus to rewrite the left-hand side of inequality (4) as follows:

$$u\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_1)\right) - u\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_0)\right) = \int_0^1 \frac{\partial u\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_t)\right)}{\partial t}\mathrm{d}t.$$

A sufficient condition for (4) is then that

$$\frac{\partial u\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_t)\right)}{\partial t} \geq \frac{\partial u\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_t)\right)}{\partial t}$$

for all $t \in [0,1]$ because integration respects monotonicity. Using the chain rule and the implicit function rule, we can rewrite this as follows:

$$\begin{aligned} & \frac{u'\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_t)\right)}{\psi'\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_t)\right)} \cdot \left[\mathbb{E}\psi(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_{a''}) - \mathbb{E}\psi(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_{a'})\right] \\ \geq \; & \frac{u'\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_t)\right)}{\psi'\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_t)\right)} \cdot \left[\mathbb{E}\psi(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_{a''}) - \mathbb{E}\psi(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_{a'})\right]. \end{aligned}$$

If ψ is Mth-degree risk-averse, the Mth-degree risk increase from $\widetilde{\varepsilon}'$ to $\widetilde{\varepsilon}''$ lowers expected utility due to Theorem 2, which in turn lowers CE. This decrease in CE raises the ratio of marginal utilities if u is more concave than ψ. In this case,

$$\frac{u'\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_t)\right)}{\psi'\left(CE(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_t)\right)} \geq \frac{u'\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_t)\right)}{\psi'\left(CE(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_t)\right)}.$$

Furthermore, if ψ is $(M+N)$th-degree risk-averse, the Nth-degree risk increase from $\widetilde{\ell}_{a''}$ to $\widetilde{\ell}_{a'}$ lowers expected utility by more when the income risk has higher Mth-degree risk. So the change from $\widetilde{\ell}_{a''}$ to $\widetilde{\ell}_{a'}$ lowers expected utility by more in the presence of $\widetilde{\varepsilon}''$ than in the presence of $\widetilde{\varepsilon}'$. This follows from the Corollary in Eeckhoudt et al. (2009) and from Ebert et al.'s (2018) results on mutual aggravation. Mathematically, we obtain

$$\mathbb{E}\psi(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_{a''}) - \mathbb{E}\psi(w_2+\widetilde{\varepsilon}''+\widetilde{\ell}_{a'}) \geq \mathbb{E}\psi(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_{a''}) - \mathbb{E}\psi(w_2+\widetilde{\varepsilon}'+\widetilde{\ell}_{a'}).$$

Nth-degree risk aversion ensures that the right-hand side is nonnegative. Combining the inequalities then shows that $U(a;\widetilde{\varepsilon}'') \succeq_I U(a;\widetilde{\varepsilon}')$ on $[\underline{a},\overline{a}]$, and Theorem 1 yields

$$\underset{a\in[\underline{a},\overline{a}]}{\arg\max}\, U(a;\widetilde{\varepsilon}'') \geq_S \underset{a\in[\underline{a},\overline{a}]}{\arg\max} U(a;\widetilde{\varepsilon}').$$

Optimal Nth-degree risk reduction increases in the strong set order following the Mth-degree risk increase of the income risk. Results (ii)-(iv) can be shown analogously.

A.5 Proof of Proposition 4

We sign the cross-derivatives of $U(s,x)$, $U(s,y)$ and $U(x,y)$ to show that the objective functions are submodular under our assumptions. The argument of CE is omitted to compress notation. For saving and self-protection we find

$$\begin{aligned} \frac{\partial^2 U(s,x)}{\partial s \partial x} &= u''(w_1-s-x) + \beta u'(CE)\frac{\partial^2 CE}{\partial x \partial x} + \beta u''(CE)\frac{\partial CE}{\partial s}\frac{\partial CE}{\partial x} \\ &= u''(w_1-s-x) + \beta u'(CE)\left[\frac{\partial^2 CE}{\partial x \partial x} - \left(-\frac{u''(CE)}{u'(CE)}\right)\frac{\partial CE}{\partial s}\frac{\partial CE}{\partial x}\right]. \end{aligned}$$

The first term is negative because u is concave. The term in square brackets is bounded by

$$\frac{\partial^2 CE}{\partial x \partial x} - \left(-\frac{\psi''(CE)}{\psi'(CE)}\right)\frac{\partial CE}{\partial s}\frac{\partial CE}{\partial x} \tag{5}$$

because u is more concave than ψ and CE is increasing in s and x. Applying the implicit function rule to CE yields:

$$\begin{aligned}
\frac{\partial CE}{\partial s} &= \frac{R\left[p(x)\psi'(c_{2L}) + (1-p(x))\psi'(c_{2N})\right]}{\psi'(CE)}, \qquad \frac{\partial CE}{\partial x} = \frac{-p'(x)\left[\psi(c_{2N}) - \psi(c_{2L})\right]}{\psi'(CE)}, \\
\frac{\partial^2 CE}{\partial s \partial x} &= -\frac{\psi''(CE)R\left[p(x)\psi'(c_{2L}) + (1-p(x))\psi'(c_{2N})\right]\cdot(-p'(x))\left[\psi(c_{2N}) - \psi(c_{2L})\right]}{\psi'(CE)^3} \\
&\quad - \frac{R(-p'(x))\left[\psi'(c_{2L}) - \psi'(c_{2N})\right]}{\psi'(CE)},
\end{aligned}$$

with c_{2L} and c_{2N} being shorthand for consumption in the loss state and the no-loss state. Direct computation then shows that (5) can be simplified to

$$-\frac{R(-p'(x))\left[\psi'(c_{2L}) - \psi'(c_{2N})\right]}{\psi'(CE)}.$$

This is nonpositive because p is decreasing and ψ is concave. As a result, $\partial^2 U(s,x)/\partial s \partial x < 0$. In the case of saving and self-insurance, we find

$$\begin{aligned}
\frac{\partial CE}{\partial y} &= \frac{-pL'(y)\psi'(c_{2L})}{\psi'(CE)}, \\
\frac{\partial^2 CE}{\partial s \partial y} &= -\frac{\psi''(CE)R\left[p\psi'(c_{2L}) + (1-p)\psi'(c_{2N})\right]\cdot(-pL'(y))\psi'(c_{2L})}{\psi'(CE)^3} - \frac{pL'(y)R\psi''(c_{2L})}{\psi'(CE)}
\end{aligned}$$

and $\partial^2 U(s,y)\partial s \partial y < 0$ follows similarly. For self-protection and self-insurance, we obtain

$$\frac{\partial^2 CE}{\partial x \partial y} = -\frac{\psi''(CE)(-p'(x))\left[\psi(c_{2N}) - \psi(c_{2L})\right]\cdot(-p(x)L'(y))\psi'(c_{2L})}{\psi'(CE)^3} - \frac{p'(x)L'(y)\psi'(c_{2L})}{\psi'(CE)}$$

and $\partial^2 U(x,y)\partial x \partial y < 0$ is obtained with similar steps.

A.6 Proof of Proposition 5

We show that $U(a_1, a_2)$ is submodular in (a_1, a_2) under our assumptions. Let $a_1'' > a_1'$ and $a_2'' > a_2'$. In the first period, concavity of u implies

$$u(w_1 - a_1' - a_2'') - u(w_1 - a_1' - a_2') \geq u(w_1 - a_1'' - a_2'') - u(w_1 - a_1'' - a_2').$$

In the second period, we would like to show that

$$u\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_{a_2''})\right) - u\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_{a_2'})\right)$$

$$\geq \quad u\left(CE(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_{a_2''})\right) - u\left(CE(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_{a_2'})\right).$$

Subscript a_j' indicates that $\widetilde{\ell}^j$ is distributed according to $F_j(\ell; a_j')$, and likewise for subscript a_j'', with $j = 1, 2$. Define $H_2(\ell; t) = tF_2(\ell; a_2'') + (1-t)F_2(\ell; a_2')$ for $t \in [0, 1]$, and let $\widetilde{\ell}^2_t$ be distributed according to $H_2(\ell; t)$. We then obtain

$$u\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_{a_2''})\right) - u\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_{a_2'})\right) = \int_0^1 \frac{\partial u\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_t)\right)}{\partial t}\, \mathrm{d}t,$$

and likewise for a_1'' instead of a_1'. It is therefore sufficient to show that

$$\frac{\partial u\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_t)\right)}{\partial t} \geq \frac{\partial u\left(CE(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_t)\right)}{\partial t}$$

for all $t \in [0, 1]$ because integration preserves monotonicity. Using the chain rule and the implicit function rule, the inequality is equivalent to

$$\begin{aligned} & \frac{u'\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_t)\right)}{\psi'\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_t)\right)} \cdot \left[\mathbb{E}\psi(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_{a_2''}) - \mathbb{E}\psi(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_{a_2'})\right] \\ \geq \quad & \frac{u'\left(CE(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_t)\right)}{\psi'\left(CE(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_t)\right)} \cdot \left[\mathbb{E}\psi(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_{a_2''}) - \mathbb{E}\psi(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_{a_2'})\right]. \end{aligned}$$

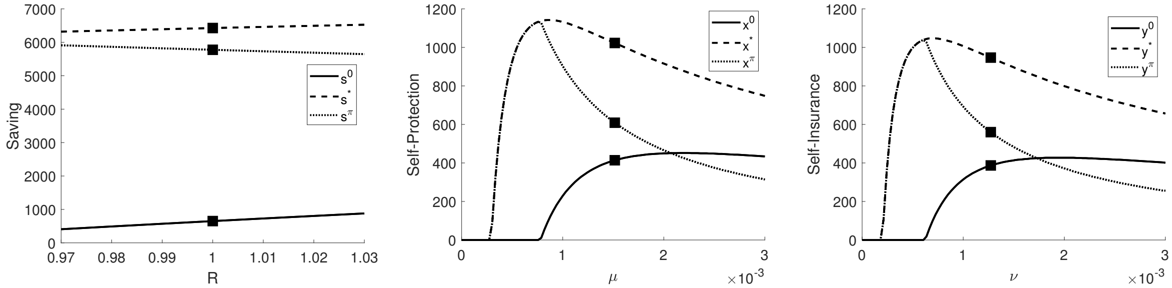
Now $w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_t$ has more N_1th-degree risk than $w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_t$, resulting in a lower CE because ψ is N_1th-degree risk-averse. This decrease in CE raises the ratio of marginal utilities if u is more concave than ψ so that

$$\frac{u'\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_t)\right)}{\psi'\left(CE(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_t)\right)} \geq \frac{u'\left(CE(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_t)\right)}{\psi'\left(CE(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_t)\right)}.$$

$(N_1 + N_2)$th-degree risk aversion ensures greater mutual aggravation, see Eeckhoudt et al. (2009) and Ebert et al. (2018). In this case, the N_2th-degree risk reduction from $\widetilde{\ell}^2_{a_2'}$ to $\widetilde{\ell}^2_{a_2''}$ increases expected utility by more when N_1th-degree risk is high rather than low, that is, in the presence of $\widetilde{\ell}^1_{a_1'}$ instead of $\widetilde{\ell}^1_{a_1''}$. As a result,

$$\mathbb{E}\psi(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_{a_2''}) - \mathbb{E}\psi(w_2 + \widetilde{\ell}^1_{a_1'} + \widetilde{\ell}^2_{a_2'}) \geq \mathbb{E}\psi(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_{a_2''}) - \mathbb{E}\psi(w_2 + \widetilde{\ell}^1_{a_1''} + \widetilde{\ell}^2_{a_2'}),$$

and N_2th-degree risk aversion ensures that the right-hand side is nonnegative. Combining the inequalities accordingly shows that $U(a_1, a_2)$ is submodular in (a_1, a_2) under our assumptions.



(a) Saving

(b) Self-protection

(c) Self-insurance

Figure 4: Effect of return parameters R, μ and ν on saving, self-protection, and self-insurance, respectively. We use the skewed income risk with $\mathbb{E}\widetilde{\varepsilon} = 0$, $\sigma(\widetilde{\varepsilon}) = \$12,500$, and $sk(\widetilde{\varepsilon}) = -1$. The squares represent values from the base case with $R = 1$, $\mu = 0.0015355$ and $\nu = 0.0012866$.

B Return parameters and precautionary behavior

Figure 4 presents the effect of the return parameters in the single-instrument cases for a skewed binary income risk with $sk(\widetilde{\varepsilon}) = -1$. Panel (a) analyzes how the gross return R affects saving behavior. Saving is increasing in R both in the presence and in the absence of income risk, corresponding to the solid and the dashed line. The effect is slightly stronger when no income risk is present. As a result, precautionary saving is decreasing in the gross return R as represented by the dotted line.

Panels (b) and (c) show how self-protection and self-insurance depend on the respective efficiency parameters μ and ν. We implement both technologies with a log-linear specification so that a higher efficiency parameter has two conflicting effects. It lowers the loss probability or loss severity for a given investment in self-protection or self-insurance, thus decreasing the need for additional use of the instrument. At the same time, a higher efficiency parameter increases the impact of additional investments thus exerting a positive effect on instrument use. This tension explains the inverted U-shapes in Panels (b) and (c) in the absence of income risk (solid line), in the presence of income risk (dashed line), and for precautionary instrument use (dotted line).

C Online Appendix

C.1 Proof of Remark 1

Second-period consumption is given by $\widetilde{c}_2 = \widetilde{w}_2 + sR + \widetilde{\ell}$ with an expected value of

$$\mathbb{E}\widetilde{c}_2 = w_2 + sR + \mathbb{E}\widetilde{\ell} = w_2 + sR - p(x)L(y).$$

It is increasing in s, x and y under our assumptions. Now let m_n denote the nth central moment of a random variable. We obtain

$$m_n\left(\widetilde{c}_2\right) = \mathbb{E}\left(\widetilde{c}_2 - \mathbb{E}\widetilde{c}_2\right)^n = \mathbb{E}\left(\widetilde{w}_2 + sR + \widetilde{\ell} - w_2 - sR - \mathbb{E}\widetilde{\ell}\right)^n = \mathbb{E}\left(\widetilde{\varepsilon} + \widetilde{\ell} - \mathbb{E}\widetilde{\ell}\right)^n$$

so that saving has no effect on any higher-order central moments. For self-protection and self-insurance, we first apply the binomial formula,

$$m_n(\widetilde{c}_2) = \sum_{k=0}^{n} \binom{k}{n} \cdot m_k(\widetilde{\varepsilon}) \cdot m_{n-k}(\widetilde{\ell}),$$

to express $m_n(\widetilde{c}_2)$ as a function of the central moments of the income risk and the loss risk. By definition, $m_0(\widetilde{\varepsilon}) = m_0(\widetilde{\ell}) = 1$ and $m_1(\widetilde{\varepsilon}) = m_1(\widetilde{\ell}) = 0$, and for $k \geq 2$ we obtain

$$m_k(\widetilde{\ell}) = p(x)(1-p(x))L(y)^k \cdot \sum_{l=1}^{k-1} \binom{k-1}{l} \cdot (-1)^{l+1} \cdot p(x)^{k-l-1}.$$

Therefore, the variance of second-period consumption is given by

$$m_2(\widetilde{c}_2) = m_2(\widetilde{\varepsilon}) + m_2(\widetilde{\ell}) = m_2(\widetilde{\varepsilon}) + p(x)(1-p(x))L(y)^2,$$

which is the sum of the variance of the income risk and the variance of the loss risk due to independence. It follows that $m_2(\widetilde{c}_2)$ is decreasing in self-protection if and only if $p(x) < 1/2$. It is always decreasing in self-insurance.

Skewness is the third standardized moment of a random variable,

$$sk(\widetilde{c}_2) = \frac{m_3(\widetilde{c}_2)}{m_2(\widetilde{c}_2)^{3/2}} = \frac{m_3(\widetilde{\varepsilon}) - p(x)(1-p(x))(1-2p(x))L(y)^3}{\left[m_2(\widetilde{\varepsilon}) + p(x)(1-p(x))L(y)^2\right]^{3/2}}.$$

It is not a simple function of the skewness of the income risk and the skewness of the loss risk. To determine the effect of self-protection on $sk(\widetilde{c}_2)$, we inspect the numerator of $\mathrm{d}sk(\widetilde{c}_2)/\mathrm{d}x$, which, after some simplifications, is given by

$$\begin{aligned} -p'(x)L(y)^2 \left[m_2(\widetilde{\varepsilon}) + p(x)(1-p(x))L(y)^2\right]^{1/2} \cdot \Big\{ & p(x)^2\left(6L(y)m_2(\widetilde{\varepsilon}) + \tfrac{1}{2}L(y)^3\right) \\ & -p(x)\left(6L(y)m_2(\widetilde{\varepsilon}) + \tfrac{1}{2}L(y)^3 + 3m_3(\widetilde{\varepsilon})\right) + \left(L(y)m_2(\widetilde{\varepsilon}) + \tfrac{3}{2}m_3(\widetilde{\varepsilon})\right) \Big\}. \end{aligned}$$

$sk(\widetilde{\varepsilon})$	0	-0.5	-1.0	-1.5	-2.0
q	0.50	0.38	0.28	0.20	0.15
ε_-	-\$12,500	-\$16,010	-\$20,225	-\$25,000	-\$30,178
ε_+	\$12,500	\$9,760	\$7,725	\$6,250	\$5,178
$\sigma(\widetilde{\delta})$	0.26	0.27	0.30	0.32	0.36

Table 9: Parameters for skewed binary income risks. We set $\mathbb{E}\widetilde{\varepsilon} = 0$, $\sigma(\widetilde{\varepsilon}) = \$12,500$ and vary $sk(\widetilde{\varepsilon})$ from 0 to -2 in decrements of 0.5. This implies unique values for q, ε_- and ε_+. We also report the standard deviation of the implied annual log earnings growth $\widetilde{\delta}$.

The sign coincides with the sign of the curly bracket, which is a quadratic function of $p(x)$. It is tedious but straightforward to show that the associated discriminant is strictly positive so there are always two zeros, denoted by p_1 and p_2. Per direct computation, one can also show that three cases are possible. If $\sigma(\widetilde{\varepsilon})sk(\widetilde{\varepsilon}) \geq \frac{2}{3}L(y)$, then $0 < p_1 < 1 \leq p_2$, and the curly bracket is positive for $p(x) < p_1$ and negative for $p(x) > p_1$; if $-\frac{2}{3}L(y) < \sigma(\widetilde{\varepsilon})sk(\widetilde{\varepsilon}) < \frac{2}{3}L(y)$, then $0 < p_1 < p_2 < 1$, and the curly bracket is positive for $p(x) \in (0, p_1) \cup (p_2, 1)$ and negative for $p(x) \in (p_1, p_2)$; if $\sigma(\widetilde{\varepsilon})sk(\widetilde{\varepsilon}) \leq -\frac{2}{3}L(y)$, then $p_1 \leq 0 < p_2 < 1$, and the curly bracket is negative for $p(x) < p_2$ and positive for $p(x) > p_2$. Remark 1 focuses on those cases where $p_1 > 0$ so that self-protection increases $sk(\widetilde{c}_2)$ for $p(x) < p_1$.

For self-insurance, the numerator of $\mathrm{d}sk(\widetilde{c}_2)/\mathrm{d}y$ is the following:

$$-3p(x)(1-p(x))L(y)L'(y)\left[m_2(\widetilde{\varepsilon}) + p(x)(1-p(x))L(y)^2\right]^{1/2}\left[(1-2p(x))L(y)m_2(\widetilde{\varepsilon}) + m_3(\widetilde{\varepsilon})\right].$$

The sign coincides with the sign of the second square bracket. It is positive if and only if

$$p(x) < \frac{1}{2}\left(1 + \frac{1}{L(y)}\sigma(\widetilde{\varepsilon})sk(\widetilde{\varepsilon})\right).$$

C.2 Parameterization of skewed income risks

To analyze the effect of downside risk on precautionary behavior, we use skewed income risks $\widetilde{\varepsilon} = [q, \varepsilon_-; (1-q), \varepsilon_+]$ in Section 5. We set $\mathbb{E}\widetilde{\varepsilon} = 0$, $\sigma(\widetilde{\varepsilon}) = \$12,500$, corresponding to 25% of annual income, and vary $sk(\widetilde{\varepsilon})$ from 0 to -2 in decrements of 0.5. We apply Ebert's (2015) Proposition 1 to find the unique $q \in (0,1)$, $\varepsilon_- < 0$ and $\varepsilon_+ > 0$ consistent with the first three moments of the income risk. Table 9 provides these parameters and also states the standard deviation of the implied log earnings growth, defined as $\widetilde{\delta} = \log(1 + \widetilde{\varepsilon}/w_2)$. The skewness of the log earnings growth coincides with the skewness of the income risk because for binary risks skewness is solely determined by the probabilities and does not depend on the outcomes.