Different Sources of the Variance of Online Consumer Ratings and their Impact on Price and Demand

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Abstract

Consumer ratings can play a decisive role in purchases by online shoppers. To examine the effect of the variance of these ratings on future product pricing and sales we develop a model which considers goods that are characterized by two types of attributes: experience attributes and experience attributes that were transformed in search attributes by consumer ratings that we call informed search attributes. For pure informed search goods where the variance in ratings is caused by an informed search attribute, we find that with increasing variance optimal price increases and demand decreases. For pure experience goods where the variance in ratings is caused by an experience attribute, we find that with increasing variance optimal price and demand decrease. For hybrid goods – where the variance in ratings is caused by both attributes – when there is low total variance, and the average rating and total variance are held constant, optimal price and demand increase as the increasing relative share of variance caused by informed search attributes increases. Via this mechanism, between two similar goods with the same average rating risk averse consumers may prefer the higher priced good with a higher variance. In addition, our model provides a theoretical explanation for the empirically observed j-shaped distribution of consumer ratings in electronic commerce.

Keywords: Consumer Rating, Variance, Experience Attribute, Informed Search Attribute, E-Commerce.
1 Introduction

When a new good is introduced on the market, consumers have uncertainty about the good’s attributes, even though these attributes may be important to make a purchase decision (Shapiro 1983). Nelson (1970) was the first to introduce the distinction between search goods and experience goods. Search goods are solely characterized by search attributes that can be determined by inspection without the necessity of use (Shapiro 1983). Examples of search goods include printing paper or blank CDs. Experience goods are solely characterized by experience attributes that can only be determined through use (Wei and Nault 2013). Examples of experience goods include digitalized books, music, or movies. Nelson (1981) generalized this dichotomous distinction and suggested that most goods can be described by search and experience attributes.

Pre-Internet and pre-Web 2.0, experience attributes were difficult to determine before purchasing a good and experiencing it: consumers relied on word-of-mouth, annual printed consumer reports, advice from friends, etc., information that was unlikely to be timely. Due to consumer ratings on e-commerce platforms, this has fundamentally changed. Online consumer ratings are most commonly provided in the form of a star rating system (indicating the valence of the consumer rating) and an optional textual review. In particular, consumer ratings offer a form of peer learning among consumers by enabling prospective consumers to learn from other consumers’ experiences (Liu et al. 2014). Thereby, they transform many experience attributes into search attributes (Hong et al. 2012), reducing the uncertainty consumers have about the good (Chen and Xie 2008, Kwark et al. 2014). We denote these attributes as informed search attributes. An example for an informed search attribute is the sound quality of a headphone set. Without additional information, assessing this attribute requires listening to the actual device – an experience attribute. This can now be inferred from online consumer ratings – an informed search attribute. However, not all experience attributes can be transformed into informed search attributes. These are quality attributes that may differ between two instances of the same good. For example, negative textual reviews for specific headphones (Amazon 2015) show that some of the headphones failed after a relatively short period of usage. From these reviews, consumers can make some
inference about the probability of failure – an informed search attribute. What they cannot infer from these reviews is whether their individual headphones will fail. Thus, the failure itself represents an experience attribute that cannot be transformed into an informed search attribute by consumer ratings.

Much of the information contained in the textual reviews is summarized in the star rating ranging from one (lowest recommendation) to five (highest recommendation) on most e-commerce websites. A bar chart shows the distribution of the star rating, with the average rating displayed prominently beneath the product name. Thus, consumers can see at a glance the average rating from other consumers (mean) and the extent to which opinions about the good differ (variance). Among the significant literature that has recently emerged on consumer ratings, several studies analyze the effect of the average consumer rating on price and demand (e.g., Chen et al. 2004, Chevalier and Mayzlin 2006, Dellarocas et al. 2007, Duan et al. 2008, Li and Hitt 2008, Luca 2011). However, only few studies explicitly analyze the effect of the variance of online consumer ratings on price and demand (Clemons at al. 2006, Hong et al. 2012, Sun 2012) and, to the best of our knowledge, none explicitly considers whether the variance is caused by informed search attributes (i.e., the sound quality of headphones) or by experience attributes (i.e., the technical failure of headphones) or by both.

Against this backdrop, we aim to answer the following research question: How does the variance of consumer ratings caused by informed search and experience attributes differentially affect price and demand?

To determine the effect of the different sources of variance of consumer ratings on price and demand we construct a model featuring a monopoly retailer and consumers that differ in taste and in risk aversion. We analyze three goods: pure informed search goods where the variance of consumer ratings is solely caused by an informed search attribute; pure experience goods where the variance of consumer ratings is solely caused by an experience attribute; and hybrid goods where the variance of consumer ratings is caused by an informed search and an experience attribute.

Our analysis yields the following main results: First, a higher variance caused by an informed search attribute always signals that a good is liked by some consumers but disliked by
others, and results in a higher price and lower demand. Second, a higher variance caused by
an experience attribute signals that there is risk associated with the good resulting in a lower
price and demand. Third, holding the average rating as well as the total variance constant
and increasing the relative share of variance caused by the informed search attribute leads
to an increase in both the price and demand for goods with low total variance. Through this
mechanism, price and demand can increase with increasing total variance. We demonstrate,
therefore, how risk averse consumers may prefer a higher priced good with a higher total
variance in ratings when deciding between two similar goods with the same average rating.
Finally, our model provides a theoretical explanation for the empirically observed j-shaped
distribution (Hu et al. 2007, 2009) of consumer ratings in e-commerce.

2 Notation and Assumptions

Our assumptions pertain to a number of different factors relating to, first, consumer het-
ergeneity, second, product characteristics and third, consumer rating behavior. These are
presented in turn.

Assumption 1 (Consumer Heterogeneity) Consumers are heterogeneous in taste and
in risk aversion. Taste and risk aversion are independent.

In line with (Sun 2012), we represent consumer taste by $\tau$ which is uniformly distributed
between zero and one, i.e. $\tau \sim U[0, 1]$. We further denote consumer risk aversion by $\theta$ which
is also uniformly distributed between zero and one, i.e. $\theta \sim U[0, 1]$. Independent tastes
and risk aversions are represented by a square with edge length 1. In Figure 1 (all figures
are listed in Appendix 2), the line segment [AB] represents taste and the line segment [AC]
represents risk aversion. For example, a consumer located in E is substantially risk averse
and the taste is slightly mismatched with the product. Our model has two periods and in
each period a unit mass of consumers is uniformly distributed within this square. To keep
the analysis simple, there is no overlap in consumers across periods.

Assumption 2 (Product Characteristics) Each product is characterized by a positive
matched quality, positive or zero mismatch costs, and a failure rate between zero and one.
Mismatch costs are limited by the matched quality of a product.
Assumption 2 defines products by three characteristics: matched quality, mismatch costs, and failure rate. 

*Matched quality* determines how much a consumer enjoys an ideal product (i.e., a product with a perfectly matched taste) that does not fail during its typical period of usage. We denote matched quality as \( v \) and assume that \( v \in \mathbb{R}^+ \). 

*Mismatch costs* are the same as in Sun (2012, p. 679) and capture “aspects of the product that would have an influence on how much consumers would differ in their enjoyment of the product”. We denote mismatch costs as \( x \) and assume that \( x \in [0, v] \). Mismatch costs are caused by informed search attributes that are perceived differently among consumers and negatively affect their enjoyment depending on their taste. Products with informed search attributes that cause mismatch costs of close to zero are a perfect fit for all consumers (i.e., typical mass market products) while products with informed search attributes that cause high mismatch costs are a perfect fit for just a small group of consumers (i.e., typical niche products). In contrast to Sun (2012), we assume mismatch costs are limited by the matched quality of a product. Thus, even consumers that maximally dislike all informed search attributes that cause mismatch costs receive non-negative enjoyment from the product if they were to obtain it for free. 

Finally, we consider the actual *product failure* as an experience attribute of the product, whereas the product’s *failure rate*, \( f \in [0, 1] \), represents an informed search attribute and accounts for the likelihood of product failure during the typical time of usage.

In the launch stage of a product, marketing communication from the manufacturer provides the dominant source of product information (Manchanda et al. 2009). This communication primarily affects first-period consumers’ purchase decisions through the reduction of uncertainty about search attributes (Narayanan et al. 2005). However, there still remains uncertainty about experience attributes. Consequently, both the consumers and the retailer have expectations of \( v, x, \) and \( f \). We denote these expectations with \( v_e, x_e \) and \( f_e \).

**Assumption 3 (Consumer Rating Behavior)** All first-period consumers publish an honest rating for the product.

Consistent with Sun (2012), we suppose that soon after a product launch all first-period consumers publish a consumer rating and there is no external manipulation of consumer ratings as discussed in Mayzlin (2006) and Luca (2011).
3 Model Analysis

We consider a two period model with a monopoly retailer and consumers. The sequence of decisions is illustrated in Figure 2.

A consumer with taste $\tau$ and risk aversion $\theta$ that purchases in the first period at price $p_1$ has expected utility $u_1$ (measured in dollars):

$$u_1 = (v_e - x_e \tau)(1 - f_e) - p_1 - f_e z \theta$$

The first part of the right hand side of (1) is equal to the expected utility of a risk neutral consumer. The last part of this equation captures a consumer’s negative utility caused by risk aversion due to potential product failure. To allow for different absolute levels of consumer risk aversion for different products, we multiply $\theta$ by a scaling factor $z \in \mathbb{R}^+$. Note that our modeling of consumer risk aversion does not make any assumptions about the specific type of risk aversion.

In the first period, a unit mass of first-period consumers enter the market. Each consumer demands at most one unit, as is the case with durable goods. The retailer sets profit maximizing price $p_1$ and consumers decide whether to purchase based on their expected utility. Consumers purchase only if their expected utility from consumption is weakly greater than zero, and receive zero utility otherwise. All consumers that decide to purchase consume the product and experience the matched quality, that is, the realized matched quality $v_r$. They also experience the mismatch costs, that is, the realized mismatch cost $x_r$. Further, they experience whether their specific product fails or not. For a consumer that purchases a product of realized matched quality $v_r$ and with realized mismatch cost $x_r$, at price $p_1$ the utility is $v_r - x_r \tau - p_1$ if the product does not fail and $-p_1$ otherwise. After learning the realizations of $v_r$ and $x_r$, each consumer publishes an honest rating $v_r - x_r \tau$ for the product if it does not fail or a rating of zero if the product fails.

In the second period, a unit mass of second-period consumers enter the market. These consumers and the retailer observe the mean and the variance of the rating distribution. Based on this information, the retailer sets profit maximizing price $p_2$ and consumers decide whether to purchase.

In the following we separately analyze pure informed search goods, pure experience goods
and hybrid goods. The failure rate for pure informed search goods is zero. For pure experience goods, the mismatch costs are equal to zero. Depending on the product type, (1) simplifies to:

\[
u_1 = \begin{cases} 
v_e - x_e \tau - p_1 & \text{for informed search goods,} \\
v_e (1 - f_e) - p_1 - f_e z \theta & \text{for experience goods,} \\
(v_e - x_e \tau) (1 - f_e) - p_1 - f_e z \theta & \text{for hybrid goods.} \end{cases}
\]

3.1 Pure Informed Search Goods

Pure informed search goods are characterized by zero failure, \( f = 0 \), and the variance of the rating distribution is caused by the informed search attribute.

**First Period:** First-period consumers make their purchase decisions based on \( v_e \) and \( x_e \). After the retailer chooses price \( p_1 \), the expected utility of a first-period consumer is

\[
u_1 = v_e - x_e \tau - p_1.
\]

Solving \( v_e - x_e \tau - p_1 = 0 \) for \( \tau \) yields the taste of the indifferent consumer which we denote with \( \tilde{\tau}_1 = (v_e - p_1) / x_e \). All first-period consumers with \( \tau \leq \tilde{\tau}_1 \) purchase, and all consumers with \( \tau > \tilde{\tau}_1 \) do not. As \( \tau \) is uniformly distributed between zero and one and there is a unit mass of potential consumers, first-period demand \( D_1 \) is equal to \( \tilde{\tau}_1 \). Consumers that purchase publish an honest product rating based on the realizations of matched quality \( v_r \) and mismatch costs \( x_r \). As tastes are uniformly distributed in \([0, D_1]\), ratings are also uniformly distributed in \([v_r - D_1 x_r, v_r]\). The average rating \( M \), and the variance of ratings \( V \) can be computed, respectively, as

\[
M = v_r - 0.5 D_1 x_r \quad \text{and} \quad V = \frac{D_1^2 x_r^2}{12}.
\]

**Second Period:** Second-period consumers learn about the product from the rating distribution. Consumers can directly derive \( v_r \) and \( x_r \) by rearranging (3):

\[
v_r = M + \sqrt{3V} \quad \text{and} \quad x_r = \frac{\sqrt{12V}}{D_1}.
\]

Thus, second-period consumers have no remaining uncertainty about informed search attributes. The utility for a second period consumer is \( u_2 = v_r - x_r \tau - p_2 \). Based on \( u_2 \) the retailer can derive the taste of the indifferent consumer as a function of the second-period
product price $p_2 : \tilde{\tau}_2 = (v_r - p_2)/x_r$. As taste is uniformly distributed among consumers, the second-period demand $D_2$ is also equal to $\tilde{\tau}_2$. Knowing this demand, the retailer can maximize profits by solving: $\max_{p_2} p_2 D_2$. This leads to the optimal second-period levels of price and demand:

$$p^*_2 = \frac{v_r}{2} \quad \text{and} \quad D^*_2 = \frac{v_r}{2x_r}.$$  

(5)

In terms of $M$ and $V$ optimal price and demand can be rewritten as:

$$p^*_2 = \frac{M}{2} + \frac{\sqrt{3V}}{2} \quad \text{and} \quad D^*_2 = \frac{D_1}{4} \left( \frac{M}{\sqrt{3V} + 1} \right).$$  

(6)

Based on these representations of $p^*_2$ and $D^*_2$, we present the effects of $M$ and $V$ on optimal price and demand for pure informed search goods in Proposition 1 (see Appendix 1 for the proofs of the propositions):

**Proposition 1**  
For pure informed search goods, price and demand both increase with the average rating, price increases and demand decreases with the variance of ratings.

The intuition behind Proposition 1 is as follows. First, a higher average rating is a credible signal of overall product quality. Thus, the retailer charges a higher price and consumers have a higher demand for a product with a higher quality. In literature the findings on the impact of average ratings on sales of pure informed search goods are equivocal. The first part of Proposition 1 represents a theoretical confirmation of the empirical findings of Chevalier and Mayzlin (2006), Sun (2012), Li and Hitt (2008), and Dellarocas et al. (2007) that found a positive impact of average consumer ratings on sales for pure informed search goods (books and movies).

Second, a high variance of product ratings indicates that the mismatch cost is relatively high. This means that consumers with tastes that closely match the product enjoy the product much more than the average rating would suggest. The retailer charges a higher price to all consumers to skim the higher willingness to pay of consumers with tastes that closely match the product. This higher price always deters some consumers with tastes that do not closely match the product. Figure 3 illustrates the response of second-period price and demand to changes in the average and the variance of ratings. In contrast to Sun (2012), we do not find that a higher variance of ratings may also increase second-period demand.
In Sun’s model, a necessary condition for such an effect is that the average rating $M$ is negative. From (3), we know that a negative average rating means that $x_r > 2v_r/D_1$. $D_1$ has a maximum of 1 which implies that $x_r > 2v_r$. This would mean that the enjoyment of a consumer with least matched taste (i.e., a consumer with $\tau = 1$) is at most $-v_r$ if $p_1 = 0$. As most products do not exhibit such characteristics, our second assumption rules out the possibility of $M$ being negative by assuming $x_r$ is non-negative, $x_r \in [0, v_r]$.

### 3.2 Pure Experience Goods

Pure experience goods are characterized by a positive probability of failure, $f > 0$, and there is no mismatch cost, $x = 0$. For these goods, the variance of the rating distribution is caused entirely by the experience attribute.

**First Period:** First-period consumers make their purchase decisions based on $v_e$ and $f_e$. After the retailer chooses a price $p_1$, the expected utility of a first-period consumer is

$$u_1 = v_e(1 - f_e) - p_1 - f_ez\theta.$$  

Solving $v_e(1 - f_e) - p_1 - f_ez\theta = 0$ for $\theta$ yields the risk aversion of an indifferent consumer which we denote by $\tilde{\theta}_1 = (v_e(1 - f_e) - p_1)/f_ez$. All first-period consumers with $\theta \leq \tilde{\theta}_1$ purchase, while all consumers with $\theta > \tilde{\theta}_1$ do not. As $\theta$ is uniformly distributed between zero and one and we have a unit mass of consumers, first-period demand $D_1$ is equal to $\tilde{\theta}_1$. Consumers who purchase publish an honest product rating based on the realization of matched quality $v_r$ and whether the purchased product fails. As mismatch costs are zero for pure experience goods, consumers publish either a rating of $v_r$ if the product does not fail, or a rating of zero if the product fails. This results in ratings of first-period consumers where $(1 - f_r)$ percent of the ratings are $v_r$ and $f_r$ percent are zero. For this rating distribution, the average rating $M$, and the variance of ratings $V$ can be computed, respectively, as

$$M = v_r(1 - f_r) \quad \text{and} \quad V = v_r^2 f_r (1 - f_r). \quad (7)$$

**Second Period:** As with pure informed search goods, second-period consumers learn about the product from the rating distribution. Second period consumers can learn about $v_r$ and $f_r$ by rearranging (7):

$$v_r = M + \frac{V}{M} \quad \text{and} \quad f_r = \frac{V}{M^2 + V}. \quad (8)$$
After deriving \( v_r \) and \( f_r \), second-period consumers have no uncertainty left about the matched quality and the failure rate of the product. However, even after learning about the failure rate, there is no guarantee that an individual product may not fail. Thus, the expected utility for a second-period consumer is 

\[ u_2 = v_r (1 - f_r) - p_2 - z f_r \theta \]

where the term \( z f_r \theta \) captures consumer risk aversion with regard to product failure. Based on \( u_2 \) the retailer can derive the risk aversion \( \tilde{\theta}_2 \) of the indifferent consumer as a function of the second-period product price \( p_2 \):

\[ \tilde{\theta}_2 = \frac{v_r (1 - f_r) - p_2}{z f_r}. \]  

Again, second-period demand \( D_2 \) is equal to \( \tilde{\theta}_2 \) and the retailer solves:

\[ \max_{p_2} p_2 D_2 \]

resulting in the optimal second-period levels of price and demand:

\[ p_2^* = \frac{v_r (1 - f_r)}{2} \]
\[ D_2^* = \frac{v_r (1 - f_r)}{2 f_r z}. \]  

In terms of M and V, second-period optimal levels of price and demand can be rewritten as:

\[ p_2^* = \frac{M}{2} \]
\[ D_2^* = \frac{M^3}{2V z} + \frac{M}{2z}. \]  

We use these representations of \( p_2^* \) and \( D_2^* \) to present the effects of \( M \) and \( V \) on optimal price and demand for pure experience goods in Proposition 2:

**Proposition 2**  
*For pure experience goods, price and demand both increase with the average rating, price is not affected by the variance of ratings, and demand decreases with an increasing variance of consumer ratings.*

As with pure informed search goods, a higher average rating acts as a credible signal of higher expected product quality for consumers and for the retailer, and therefore increases price and demand. We find that the variance of product ratings does not affect the price and always has a negative effect on demand. The intuition for this result is as follows: First, given a constant average rating, a higher variance of ratings implies both a higher matched quality and a higher failure rate so that the expected utility of a risk neutral consumer remains constant. As consumers in our model are risk averse, their expected utility decreases with an increasing variance of product ratings. At the same time, the retailer sets the optimal price as if all consumers were risk neutral because the additional revenue from increased sales to consumers with high risk aversion due to a lower price is always lower than the lost revenue from consumers with a lower risk aversion. Given that price does not depend on
the variance of consumer ratings and the expected utility of risk averse consumers decreases with an increasing variance of product ratings, it follows naturally that demand decreases with increasing variance of consumer ratings. Figure 4 illustrates the response of optimal price and demand to changes in the variance of product ratings.

3.3 Hybrid Goods

Hybrid goods are characterized by a positive probability of failure, \( f > 0 \), and positive mismatch costs, \( x > 0 \). For these goods, the variance of consumer ratings depends on both the informed search and the experience attributes.

**First Period:** First-period consumers make their purchase decisions based on \( v_e, x_e \), and \( f_e \). After the retailer chooses price \( p_1 \), the expected utility of a first-period consumer is
\[
  u_1 = (v_e - x_e \tau)(1 - f_e) - p_1 - f_e z \theta.
\]
Given \( u_1 \) and the independence of taste and risk aversion, we can derive first-period demand \( D_1 \). First, we need to derive the taste of an indifferent consumer with zero risk aversion, \( \tau^\theta=0 \), and the risk aversion of an indifferent consumer given that taste is zero, \( \theta^\tau=0 \). Due to the independence, first-period demand is equal to the area of the triangle \([A, \tau^\theta=0, \theta^\tau=0] \) (see Figure 5 for an example) with
\[
  \tau^\theta=0 = (p_1 + v_e (f_e - 1))/(x_e (f_e - 1)),
\]
and
\[
  \theta^\tau=0 = (v_e (1 - f_e) - p_1)/z f_e).
\] Thus, \( D_1 = 0.5 \tau^\theta=0 \theta^\tau=0 \).

Consumers that purchase publish an honest rating based on \( v_r, x_r \), and whether the purchased product fails. They publish a rating \( r = v_r - x_r \tau \) if the product does not fail and a rating of \( r = 0 \) if it does. This results in ratings of first-period consumers, where \((1 - f_r)\) percent of the ratings are \( v_r - x_r \tau \) and \( f_r \) percent are zero. For products that do not fail, ratings are triangularly distributed between \( v_r - \tau^\theta=0 x_r \) and \( v_r \) with mode at \( v_r \). Figure 6 illustrates the rating distribution for hybrid goods. This distribution has the typical j-shape which has been found for almost all products sold on amazon.com (Hu et al. 2007, 2009).

In contrast to pure informed search goods and pure experience goods, the enjoyment of hybrid goods depends not only on two, but on three product characteristics. Thus, it is not sufficient to consider only the average and the variance of ratings to derive the relevant product characteristics from the rating distribution. To distinguish if a mediocre rating and a positive variance is caused by informed search attributes (mismatch costs),
experience attribute (product failure), or a combination of the two, we decompose the total variance into (1) variance caused by mismatch costs and (2) variance caused by product failure. Variance caused by mismatch costs, denoted as $V_m$, can be derived by disregarding all negative ratings which are caused by product failure and computing the variance of the remaining rating distribution, i.e., the triangle on the right in Figure 6. As we only have two sources of variance, the variance caused by product failure, denoted as $V_f$, must be equal to the difference between the total variance and the variance caused by mismatch costs. $M$, $V_m$, and $V_f$ can be computed, respectively, as:

$$M = (v_r - \frac{\bar{\tau}_{\theta=0} x_r}{3})(1 - f_r), \quad V_m = \frac{(\bar{\tau}_{\theta=0})^2 x_r^2(1 - f_r)}{18}, \quad V_f = \frac{(1 - f_r) f_r (3v_r - \bar{\tau}_{\theta=0} x_r)^2}{9}. \quad (11)$$

**Second Period:** Based on $M$, $V_m$, and $V_f$ consumers learn about the product. Consumers can derive $v_r$, $x_r$, and $f_r$ for hybrid goods by rearranging (11):

$$v_r = M + \frac{V_f + (2V_m (M^2 + V_f))^2}{M}, \quad x_r = \frac{(2V_m (M^2 + V_f))^2}{M\bar{\tau}_{\theta=0}}, \quad \text{and} \quad f_r = \frac{V_f}{M^2 + V_f}. \quad (12)$$

After deriving $v_r$, $x_r$, and $f_r$ consumers are left with no uncertainty about the product’s informed search attributes. However, even if consumers know the exact failure rate of the product, they cannot know whether their individual product will fail. Thus, the expected utility for a second period consumer is $u_2 = (v_r - x_r \tau) (1 - f_r) - z f_r \theta - p_2$ where the term $z f_r \theta$ captures the risk of product failure. Based on the utility, the retailer can derive second-period demand $D_2$ which is equal to $0.5 \bar{\tau}_{\theta=0} x_r$.

In terms of $v_r$, $x_r$, and $f_r$, second-period demand can be written as:

$$D_2 = \frac{(v_r(1 - f_r) - p_2)^2}{2 f_r x_r z (1 - f_r)}. \quad (13)$$

Based on second-period demand the retailer solves max $p_2 D_2$ and optimal second-period price and demand can be derived as:

$$p_2^* = \frac{v_r (1 - f_r)}{3}, \quad D_2^* = \frac{2v_r^2 (1 - f_r)}{9 f_r x_r z}. \quad (14)$$

Using the relationship between $v_r$, $x_r$, and $f_r$ and $M$, $V_m$, and $V_f$, optimal price and demand can be rewritten as functions of $M$, $V_m$, and $V_f$:

$$p_2^* = \frac{M}{3} + \frac{M \sqrt{2V_m (M^2 + V_f)}}{3 (M^2 + V_f)}, \quad D_2^* = \frac{M \bar{\tau}_{\theta=0} \sqrt{2 (M^2 + V_f)}}{27V_f \sqrt{V_m z}}. \quad (15)$$
Based on these representations of $p_2^*$ and $D_2^*$, we derive the effects of the average rating, variance caused by mismatch costs, and variance caused by product failure on optimal price and demand in the following propositions.

**Proposition 3** For hybrid goods, price and demand both increase with the average rating.

Proposition 3 represents a theoretical confirmation of the empirical findings of Luca (2011) that found increases in the average rating of restaurants increases sales. A restaurant is a typical example of a hybrid good as consumer ratings on the restaurant’s atmosphere, service and food reduce the uncertainty consumers have. However, the service and food quality may differ between two visits of the same restaurant.

**Proposition 4** For hybrid goods, price increases and demand decreases with the variance caused by mismatch costs.

The intuition of Proposition 4 is the same as for pure informed search goods (see Proposition 1). Figure 7 illustrates the relationship between, on the one hand, price and demand, and on the other, the variance caused by mismatch costs.

**Proposition 5** For hybrid goods, the price decreases with variance caused by product failure. Demand decreases with variance caused by product failure if the variance caused by product failure is sufficiently low. Demand increases with variance caused by product failure if the variance caused by product failure is sufficiently high and the variance caused by mismatch costs is sufficiently low.

As for pure experience goods, a higher variance of ratings caused by product failure indicates a higher failure rate and the retailer sets the price as if all consumers were risk neutral. However, due to the positive mismatch costs, and differently from pure experience goods, the utility of a risk neutral consumer is slightly decreasing with increasing $V_f$. Thus, the optimal price always decreases if $V_f$ increases. If $V_f < 2M^2$, then increasing $V_f$ always leads to a decrease in demand. As a higher variance caused by product failure is associated with a higher failure rate, consumers are risk averse, and the product is priced as if consumers
were risk neutral, which is an intuitive result. If $V_f > 2M^2$, then increasing $V_f$ leads to an increase in demand if $V_m < (4M^63M^2V_f^2 + V_f^3)/(8M^4 + 8M^2V_f + 2V_f^2)$.

This counterintuitive finding is attributable to the necessary increase in $v$, $x$ and $f$ caused by the increased variance due to product failure. Ceteris paribus, increasing $V_f$ is associated with an increasing failure rate, and, due to the constant average rating, an increasing matched quality of the product. At the same time, a higher failure rate implies that only a smaller fraction of all sold products do not fail and, therefore, can cause variance due to consumer taste. Thus, increasing $V_f$ is also associated with an increase in $x$. This combination in connection with a decreasing price may, in a few situations, lead to an increase of demand. In a typical 5 star rating system with a rating of one indicating the worst and a rating of five indicating the best possible quality, it is not possible that $V_f > 2M^2$. Figure 8 illustrates the relationship between optimal price and demand, and variance caused by failure for a product with $V_f \leq 2M^2$.

**Proposition 6** For hybrid goods, holding the total variance constant, price always increases (decreases) with an increasing relative share of variance caused by mismatch costs (product failure). Demand increases (decreases) with an increasing share of variance caused by mismatch costs (product failure) if the total variance is sufficiently low and decreases (increases) with an increasing share of variance caused by mismatch costs (product failure) if the total variance is sufficiently high.

The intuition for Proposition 6 is as follows. Holding the total variance constant, a larger relative share of variance caused by the informed search attribute is necessarily associated with a smaller relative share of variance caused by product failure. Based on this information, the retailer increases the price to take advantage of the consumers with a higher willingness to pay. At the same time, the decreased $V_f$ indicates both a lower matched quality and a lower failure rate. This leads to a further increase of the optimal price as the utility of a risk neutral consumer increases with decreasing $V_f$ for hybrid goods.

Holding the average rating constant, a lower matched quality and a lower failure rate makes the product more attractive to risk averse consumers. This increase overcompensates for the decrease in second-period demand due to the increased product price discussed in
the paragraph above if $V < V$ or $V \geq V$ and $V_f < V_T$. If $V > V$ or $V \geq V$ and $V_f > V_T$, the positive effect of the lower failure rate on demand is smaller than the negative effect caused by the price increase due to the higher share of $V_m$. Thus, in these cases, the total effect of an increasing share of $V_m$ on demand is negative. Figure 9 illustrates the response of optimal price and demand to changes in the composition of the variance of consumer ratings for $V < V$ in (a), $V \leq V \leq V$ in (b) and $V > V$ in (c). Through the mechanism described in Proposition 6, price and demand can increase with total variance of product ratings.

Example: The shaded area in Figure 10 illustrates demand for a product with an average rating of 4, $\hat{\tau}_{\theta} = 1$, a total variance of ratings between one and two, and varying relative shares of variance caused by mismatch costs and product failure. Ceteris paribus, an increasing relative share of variance caused by mismatch costs always leads to an increase in demand as for this combination of $M$, $\hat{\tau}_{\theta} = 0$, and $V$, $V < V$. Thus, the lower bound of the shaded area represents demand for products with the lowest possible relative share of $V_m$ and the upper bound demand for products with the highest possible relative share of $V_m$.

The point marked with A represents a product with a total variance of 1.1 with approximately 70% of this variance caused by mismatch costs and 30% by product failure. This combination results in an optimal price and demand of respectively 1.75 and 0.18. As demand is increasing from bottom to top and the total variance is increasing from left to right, demand for all products at the top right of A is higher than the demand for A even if the variance of ratings for these products is also higher than the variance of A.

The solid black line in Figure 10 represents all products with the same optimal price compared to the product marked in A ($p^*_2 = 1.75$). Because the relative share of variance caused by mismatch costs increases from bottom to top, optimal price also increases in this line. Compared to the optimal price of A this results in higher prices for all products above the solid line. Thus, holding the average rating constant and increasing the total variance of ratings, we find higher optimal prices and higher demand for products at the top right of A. Comparing the worst possible variance composition marked with B ($D_2^* = 0.17$, $p^*_2 = 1.71$) with the best possible variance composition marked with C ($D_2^* = 0.34$, $p^*_2 = 1.98$) shows that product C with twice the total variance is 15% more expensive, and has twice as much.
demand compared to product B. This comparison illustrates that the effect of the variance of product ratings on product prices and sales substantially depends on the source of variance.

4 Conclusions

Previous literature that analyzes the role of the variance of consumer ratings concentrated on products where the variance is solely caused by informed search attributes. We propose a model where two product attributes may cause the variance of consumer ratings: an informed search attribute and an experience attribute. For pure informed search goods we find that with increasing variance optimal price increases and demand decreases. For pure experience goods we find that with increasing variance optimal price and demand decrease. For hybrid goods when there is low total variance, and the average rating and total variance are held constant, optimal price and demand increase as the relative share of variance caused by informed search attributes increases. Via this mechanism, between two similar goods with the same average rating risk averse consumers may prefer the higher priced good with a higher variance. In addition, our model provides a theoretical explanation for the empirically observed j-shaped distribution of consumer ratings in e-commerce.

Our findings have important managerial implications: First, retailers could improve their sales forecasts and increase profits by charging higher prices for products for which a relatively larger share of the variance is caused by informed search attributes. Second, they could implement mechanisms to explicitly communicate information about the composition of the variance for decision making of customers.

Our study also suggests several directions for future research. First, our model generates testable predictions regarding the effect of the variance of consumer ratings on product price and consumer demand. The sign of this effect depends to a large degree on the source of this variance. This provides an opportunity for empirical and experimental investigations into the effects of the variance of consumer ratings that consider the different sources of variance. Second, our results suggest that consumers would benefit from having information about the composition of the variance of product ratings. Future research may develop semantic techniques (e.g., as in Archak et al. 2011) to identify the respective shares of variance caused by mismatch costs or by product failure.
References


Appendix 1: Proofs

Proof of Proposition 1. Differentiating the optimal price and demand for pure informed search goods with respect to $M$ and $V$ gives

\[
\frac{\partial p^*_2}{\partial M} = \frac{1}{2}, \quad \frac{\partial p^*_2}{\partial V} = \frac{3}{4\sqrt{3V}}, \quad \frac{\partial D^*_2}{\partial M} = \frac{D_1}{4\sqrt{3V}}, \text{ and } \frac{\partial D^*_2}{\partial V} = -\frac{3MD_1}{8(3V)^{3/2}}.
\]

Recall that $M$, $V$, and $D_1$ are positive by definition. Thus, we have

\[
\frac{\partial p^*_2}{\partial M} > 0, \quad \frac{\partial D^*_2}{\partial V} > 0, \quad \frac{\partial D^*_2}{\partial M} > 0 \text{ and } \frac{\partial D^*_2}{\partial V} < 0.
\]

Q.E.D.

Proof of Proposition 2. Differentiating the optimal price and demand for pure experience goods with respect to $M$ and $V$ gives

\[
\frac{\partial p^*_2}{\partial M} = \frac{1}{2}, \quad \frac{\partial p^*_2}{\partial V} = 0, \quad \frac{\partial D^*_2}{\partial M} = \frac{3M^2 + V}{2Vz}, \text{ and } \frac{\partial D^*_2}{\partial V} = -\frac{M^3}{2V^2z}.
\]

As $M$, $V$, and $z$ are positive by definition, we have

\[
\frac{\partial p^*_2}{\partial M} > \frac{1}{2}, \quad \frac{\partial p^*_2}{\partial V} = 0, \quad \frac{\partial D^*_2}{\partial M} > 0, \text{ and } \frac{\partial D^*_2}{\partial V} < 0.
\]

Q.E.D.

Proof of Proposition 3. Differentiating and rearranging optimal price and demand for hybrid goods with respect to $M$ yields

\[
\frac{\partial p^*_2}{\partial M} = \left(\sqrt{2V_mV_f}\right) \left(3 \left(M^2 + V_f\right)^{3/2}\right) + 1/3,
\]

and

\[
\frac{\partial D^*_2}{\partial M} = \left(\tilde{\tau}_1/\sqrt{27V_f}\right) \left(\sqrt{2(M^2 + V_f)/V_m} + 2\right) \left(V_f + 4M^2 + \sqrt{2V_f(M^2 + V_f)} + M^2\sqrt{(2V_m)/(M^2 + V_f)}\right).
\]

As $M$, $V_m$, $V_f$, $\tilde{\tau}_1$, and $z$ are positive by definition, we have

\[
\frac{\partial p^*_2}{\partial M} > 0, \quad \text{and } \quad \frac{\partial D^*_2}{\partial M} > 0.
\]

Q.E.D.
Proof of Proposition 4. Differentiating the optimal price and demand with respect to $V_m$ yields
\[
\frac{\partial p_2^*}{\partial V_m} = \frac{\sqrt{2}M}{6 \left(V_m (M^2 + V_f)\right)^{1/2}}
\]
and
\[
\frac{\partial D_2^*}{\partial V_m} = -\left(\frac{M\tilde{\tau}^{\theta=0}_1 (2V_m (M^2 + V_f))^{1/2} (M^2 + V_f 2V_m)}{(54V_f V_m^2)}\right).
\]

As $M$, $V_m$, and $V_f$ are positive by definition, we have $\frac{\partial p_2^*}{\partial V_m} > 0$. The sign of $\frac{\partial D_2^*}{\partial V_m}$ solely depends on $(M^2 + V_f - 2V_m)$ which is positive if $V_m > \frac{M^2}{2} + \frac{V_f}{2}$. From Assumption 1 we have $x \leq v$. Rewriting this inequality in terms of $M$, $V_f$, and $V_m$ and simplifying leads to: $V_m < \frac{\tilde{\tau}^{\theta=0}_1 (2(V_f + V_m))^{1/2} (M^2 + V_f 2V_m)}{2(\tilde{\tau}^{\theta=0}_1 - 3)^2}$. As $\tilde{\tau}^{\theta=0}_1 \in [0, 1]$, this contradicts $V_m > \frac{M^2}{2} + \frac{V_f}{2}$. Thus, $\frac{\partial D_2^*}{\partial V_m} < 0$. Q.E.D.

Proof of Proposition 5. Differentiating optimal price and demand with respect to $V_f$ gives
\[
\frac{\partial p_2^*}{\partial V_f} = -\frac{M^2 V_m}{3(M^2 + V_f)^2 \sqrt{(2M^2 V_m)(M^2 + V_f)^{-1}}}
\]
and
\[
\frac{\partial D_2^*}{\partial V_f} = -\frac{\tilde{\tau}^{\theta=0}_1 \left(8M^3 V_m + \sqrt{(2M^2 V_m)((M^2 + V_f)^{-1})} (2M^4 - V_f^2 + 2V_f V_m + M^2 V_f + 4M^2 V_m)\right)}{54V_f V_m^2}.
\]

As $M$, $V_m$, and $V_f$ are positive by definition, we have $\frac{\partial p_2^*}{\partial V_f} < 0$. The sign of $\frac{\partial D_2^*}{\partial V_f}$ depends on the sign of the term in parenthesis. If this term is positive, then we have $\frac{\partial D_2^*}{\partial V_f} < 0$, if it is negative, then $\frac{\partial D_2^*}{\partial V_f} > 0$. A necessary condition that the term becomes negative is that $V_f > 2M^2$ as $(2M^4 - V_f^2 + 2V_f V_m + M^2 V_f + 4M^2 V_m)$ is always positive if $V_f < 2M^2$.

Assuming that $V_f > 2M^2$ and solving
\[
8M^3 V_m + \left(\frac{(2M^2 V_m)}{(M^2 + V_f)}\right)^{1/2} (2M^4 - V_f^2 + 2V_f V_m + M^2 V_f + 4M^2 V_m) = 0
\]
for $V_m$ gives
\[
V_m = \left((M^2 + V_f) \left(-2M^2 + V_f\right)^2 / (2(2M^2 + V_f)^2).\right)
\]
As
\[
8M^3 V_m + \left(\frac{(2M^2 V_m)}{(M^2 + V_f)}\right)^{1/2} (2M^4 - V_f^2 + 2V_f V_m + M^2 V_f + 4M^2 V_m)
\]

20
is strictly increasing in $V_m$, we have $\frac{\partial D^*_m}{\partial V_f} > 0$ if $V_f > 2M^2$ and

$$V_m < \left( (M^2 + V_f) \left( -2M^2 + V_f \right)^2 \right) / \left( 2 \left( 2M^2 + V_f \right)^2 \right)$$

Q.E.D.

**Proof of Proposition 6.** To analyze the effect of the relative share of $V_f$ (which is equivalent to the effect of the relative share of $V_m$), we need to substitute $V_m$ by $V - V_f$ in (15). Differentiating the resulting optimal price and demand with respect to $V_f$ and rearranging terms it gives

$$\frac{\partial p^*_f}{\partial V_f} = -\frac{\sqrt{2M^2(M^2+V)}}{6(M^2+V_f)^2 \sqrt{(M^2(V-V_f))(M^2+V_f)}}, \quad \text{and} \quad \frac{\partial D^*_m}{\partial V_f} = \frac{\sqrt{2M^2V(x_V)^2}}{54V^2z\sqrt{(V-V_f)(M^2+V_f)}}((M^2V_f + V^2 + \frac{2V_f(V-V_f)^2}{M^2+V_f})-(V-V_f)(4(V-V_f) + 2M^2 + 4M^2\sqrt{2V-V_f(M^2+V_f)})) \quad \text{As} \quad V_f \quad \text{is by definition always smaller than V, we have} \quad \frac{\partial p^*_f}{\partial V_f} < 0 \quad \text{and, vice versa,} \quad \frac{\partial p^*_f}{\partial V_m} > 0. \quad \text{As} \quad \frac{\sqrt{2M^2V(x_V)^2}}{54V^2z\sqrt{(V-V_f)(M^2+V_f)}} \quad \text{is always positive, the sign of} \quad \frac{\partial D^*_m(V,M,V_f,\tau_1^0,z)}{\partial V_f} \quad \text{depends only on the term:}

$$(M^2V_f + V^2 + \frac{2V_f(V-V_f)^2}{M^2+V_f}) - (V-V_f)(4(V-V_f) + 2M^2 + 4M^2\sqrt{2V-V_f(M^2+V_f)}) \quad (16)$$

which is strictly increasing in $V_f$ for $V_f \in [0,V]$ and strictly decreasing in $V$. From our assumptions that $x < v$ and $\tau_1^0 < 1$ we get $V - \frac{(\tau_1^0)^2(M^2+V)}{3(\tau_1^0)^2+4\tau_1^0+6} < V_f < \frac{4V-2M^2(\tau_1^0)^2-36V\tau_1^0+81V}{6(\tau_1^0)^2-36\tau_1^0+81}$. Inserting the upper bound of $V_f$ into (16) and solving (16) = 0 for $V$ gives $V = \overline{V}$. Thus, if $V < \overline{V}$, we have $\frac{\partial D^*_m}{\partial V_f} < 0$ and, vice versa $\frac{\partial D^*_m}{\partial V_m} > 0$. Inserting the lower bound of $V_f$ into (16) and solving (16) = 0 for $V$ gives $\overline{V}$. Thus, if $V > \overline{V}$, we have $\frac{\partial D^*_m}{\partial V_f} > 0$, and, vice versa $\frac{\partial D^*_m}{\partial V_m} < 0$. If $V \geq \overline{V}$ and $V \leq \overline{V}$, the sign of (16) depends on the specific value of $V_f$. As (16) is strictly increasing with increasing $V_f$, and (16) is neither always positive nor always negative there is some threshold $V_T$ where $\frac{\partial D^*_m}{\partial V_f} < 0$, and $\frac{\partial D^*_m}{\partial V_m} > 0$ if $V_f < V_T$ and $\frac{\partial D^*_m}{\partial V_f} > 0$, and $\frac{\partial D^*_m}{\partial V_m} < 0$ if $V_f > V_T$. Q.E.D.
Appendix 2: Figures

Figure 1: Consumer Taste and Risk Aversion

1. Product is launched and the retailer sets a profit maximizing first-period price for the product based on its expectations about product characteristics
2. First-period consumers decide whether to purchase based on the first-period price and their expectations about product characteristics
3. First-period consumers post honest ratings of the product on the retailer’s website
4. Retailer observes consumer ratings and sets profit maximizing price for second-period consumers
5. Second-period consumers observe consumer ratings and second-period price set by the retailer and decide whether to buy the product

Figure 2: Sequence of decisions
Figure 3: Optimal Second-Period Price and Demand for Pure Informed Search Goods

Figure 4: Optimal Second-Period Price and Demand for Pure Experience Goods

Figure 5: First-Period Demand for Hybrid Goods
Figure 6: Rating Distribution for Hybrid Goods

Figure 7: Optimal Price and Demand for Hybrid Goods - Variance caused by Mismatch Costs

Figure 8: Optimal Price and Demand for Hybrid Goods - Variance caused by Product Failure
Figure 7: Equilibrium Price and Demand for Observational Search and Experience Products

(a) (b) (c)

Figure 8: Equilibrium Price and Demand Observational Search and Experience Products

Figure 9: Optimal Price and Demand for Hybrid Goods - Changes in the Composition of the Variance

Figure 10: Optimal Demand for Products with Different Variance Compositions