Interaction of Debt Agency Problems and Optimal Capital Structure: Theory and Evidence

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Abstract

Does more leverage always worsen the debt agency problem? This paper presents a unified analysis that accounts for both risk-shifting and under-investment debt agency problems. For firms with positive marginal volatility of investment (defined as the change in cash flow volatility corresponding to a change of investment scale), equity holders’ risk-shifting incentive will mitigate the under-investment problem. This implies that, contrary to conventional views, the total agency cost of debt does not uniformly increase with leverage. This model further predicts that, for high growth firms in which the under-investment problem is severe, the optimal debt ratio is positively related to the marginal volatility of investment. Empirical results support this prediction.

1. Introduction

Two principal debt agency problems have been identified in the literature. One is the risk-shifting or asset substitution problem, first identified by Jensen and Meckling (1976). In their views, equity can be seen as a call option on the firm and call values increase with the volatility of the underlying asset. This creates an incentive for equity holders to shift the firm’s investments into high risk projects. The other is the under-investment incentive. Myers (1977) argues that, when a firm’s leverage increases, equity holders have an incentive to under-invest in positive net present value (NPV) projects. This occurs because equity holders bear the costs of investment but capture only part of the net benefit, and the rest accrues to bond holders. Rational debt holders are aware of the equity holders’ incentives to shift risk and under-invest, thereby pricing the debt accordingly and demanding a higher rate of return. Thus, the adverse consequences of debt agency problems

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are entirely borne by the equity holders themselves through the increased cost of debt financing.

The implications of debt agency problems for an optimal capital structure have been widely studied in the literature. In their seminal paper, Jensen and Meckling (1976) propose that a firm's optimal debt-equity ratio is achieved by equating the marginal agency cost of debt and the marginal agency cost of equity. This theory assumes that the agency cost of debt increases monotonically with the amount of leverage the firm employs. The relationship between the leverage ratio and debt agency cost has been explored by several studies, but previous research typically focuses on only one type of the debt agency problem, either the risk-shifting or the under-investment problem. Gavish and Kalay (1983) adopt a very simple one-period model to examine the relationship between leverage and the severity of the risk-shifting problem. They argue that equity holders' risk-shifting incentive is not an increasing function of the leverage ratio. Green and Talmor (1986) restate the problem in a similar model context, and find that Gavish and Kalay's original conclusion is a misinterpretation of the result. Their study suggests that more debt aggravates shareholders' incentive to take risk. This is consistent with the original conjecture of Jensen and Meckling. Similarly, Myers (1977) also demonstrates a positive relationship between leverage and the agency cost of under-investment.

Given the properties of the risk-shifting and under-investment problems examined in isolation, it is often asserted that more leverage exacerbates the debt agency problem. Such an assertion assumes that the two debt agency problems drive firms' investment decisions in the same direction, and neglects the potential interaction between the two. Suppose that the volatility of a firm's cash flows increases with investment scale, or that a firm faces a potential project with cash flows positively correlated with the cash flows from its existing assets. In this case, increasing the scale of investment would increase the total volatility of the firm's cash flows. In a leveraged firm, equity holders would increase investment to increase firm risk, while their under-investment incentives discourage them from investing. Therefore, the two incentive problems will affect the firm's investment and debt policy in different directions.

The potential interaction between the under-investment problem and the risk-shifting incentive has not been completely ignored. Myers (1977) points out that, if investment increases the variance of project return, equity holders' incentive to shift risk will mitigate the under-investment problem. He concludes: "The impact of risky debt on the market value of the firm is less for firms holding investment options on assets that are risky relative to the firms' present assets. In this sense we may observe risky firms borrowing more than safe ones" (Myers (1977), p. 167). Myers' argument suggests a positive relationship between the optimal debt level and business risk. Despite several attempts to test this idea, however, the empirical evidence is ambiguous (e.g., see Long and Malitz (1985), Bradley, Jarrell, and Kim (1984), and Titman and Wessels (1988)). I argue that

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1 As Jensen and Meckling comment. "We are fairly confident of our arguments regarding the signs of the first derivatives of the functions ..." (Jensen and Meckling (1976), p. 346). Here "the functions" refer to the debt agency cost function and the equity agency cost function.
this is because the interaction between the two debt agency problems has not been fully understood.

To address this lack, I develop a simple model that captures both the risk-shifting and the under-investment problems. In the model, a firm faces a discretionary investment decision, and the terminal firm value is a random variable whose mean and volatility both depend on the size of the investment. Thus the risk-shifting and under-investment incentives will affect the investment decision of a leverage firm in the same or different direction, and the total agency cost of debt will depend on the tradeoff between the two incentive problems.

A key result of the model is that, if the volatility of project cash flows increases with investment scale, risk-shifting by equity holders will mitigate the under-investment problem. This implies that, contrary to the conventional wisdom, the total agency cost of debt is not monotonically increasing with leverage. Furthermore, the model predicts a unique relationship between the optimal debt level and the marginal volatility of investment (MVI), which I define as the change of cash flow volatility corresponding to a change of investment scale. For high growth firms, the optimal level of debt increases with the magnitude of MVI when MVI is positive, but declines with the magnitude of MVI when MVI is negative. It suggests a positive relationship between the optimal debt level and MVI. For low growth firms, the optimal level of debt decreases with the magnitude of MVI for both positive and negative MVI firms. It suggests a negative relationship between the optimal debt level and MVI when MVI is positive, and a positive relationship between the optimal debt level and MVI when MVI is negative.

I test the model’s predictions using cross-sectional data from Compustat (1976–1995). Empirical findings support these predictions. For high growth firms, I find a significant positive relationship between leverage and MVI. For low growth firms, I observe a significantly negative relationship between leverage and MVI when MVI is positive, and a weakly positive relationship between the two when MVI is negative.

The paper is organized as follows. Section II presents a one-period discrete-time model that captures both the risk-shifting and under-investment problems. Section III reviews and interprets empirical tests and results. Section IV concludes the paper.

II. The Model

I examine the effects of leverage on a firm’s investment decision in a one-period discrete-time model. To highlight the central conflicts of interest between equity holders and bond holders, I abstract from many important influences on financial decisions, such as risk aversion, dividend policy, taxes, agency costs of equity, and deadweight costs of bankruptcy. The model, which allows for a simultaneous analysis of the risk-shifting and under-investment problems, is described by the following assumptions:
i) Equity holders control the firm, and the managerial objective is to maximize the value of equity.²

ii) There are no taxes or bankruptcy costs, and there is no information asymmetry between insiders and outside investors.

iii) Investment policy cannot be established (or specified) contractually ex ante.

iv) All economic agents are risk neutral, and the risk-free rate is zero.

v) Investment decisions are made at time 0, and the firm liquidates at time 1.

Consider a firm with a single investment project. Let

\[ \hat{X}_1 = \mu(k_0) + \sigma(k_0)\varepsilon_1, \]

where \( \hat{X}_1 \) is the end-of-period cash flow at time 1, which is also the terminal value of the firm; \( k_0 \) measures the investment scale and is chosen by equity holders at time 0; \( \varepsilon_1 \) is a random variable that represents the state of nature at time 1 resulting from randomness in technology, in the prices of inputs and outputs, and in the demand facing the firm. I assume that \( \varepsilon_1 \) is a random variable with continuous density function \( f(\varepsilon_1) \), a mean of zero, and a variance of one. Given equation (1), the terminal firm value, \( \hat{X}_1 \), is a random variable whose mean and volatility both depend on the initial investment, \( k_0 \). To ensure limited liability for equity holders, I assume that \( \hat{X}_1 = \mu(k_0) + \sigma(k_0)\varepsilon_1 \geq 0 \) for all \( \varepsilon_1 \) and \( k_0 \). This implies that

\[ \varepsilon_1 > \text{min} \left[ -\frac{\mu(k_0)}{\sigma(k_0)} \right] = \varepsilon^*, \]

where \( \varepsilon^* \) is the low bound of \( \varepsilon_1 \).

I assume that the mean function of the firm's cash flows, \( \mu(k_0) \), is a twice continuously differentiable function with \( \mu''(k_0) > 0 \) and \( \mu'''(k_0) < 0 \). This is consistent with many types of technologies, such as a Cobb-Douglas production function. I assume that \( \sigma(k_0) \) is once differentiable in \( k_0 \), and I call its first derivative \( [\sigma'(k_0)] \) the marginal volatility of investment (MVI). The sign on \( \sigma'(k_0) \) is unrestricted, since it may differ across firms or industries. In fact, the relationship between the volatility of cash flows and the investment scale may depend on a firm's investment opportunity set and the stochastic driving forces of cash flows.

I assume that, before time 0, the firm issues equity and debt to raise capital for investment in the project. Suppose it issues zero-coupon bonds that have a total face value of \( D \) and mature at time 1. As I have assumed above, monitoring and bonding costs exclude the possibility of contracting equity holders to a particular \( k_0 \) ex ante. Therefore, at time 0, for a given level of debt, \( D \), equity holders would invest \( k_0 \) units of capital so as to maximize the net present value of their claims.³

²This is an important assumption to cast the conflicts of interest between equity holders and debt holders. In fact, managers may pursue their own interests instead of those of equity holders. There are numerous studies of the agency costs of managerial discretion, e.g., Stulz (1990). Along another line of the research on optimal capital structure under asymmetric information, Dybvig and Zender (1991) argue that the optimal choice of managerial compensation contract may resolve the conflicts of interest among different classes of stakeholders of firm. These are interesting issues, but beyond the scope of this study.

³As in Myers (1977), Long and Malitz (1985), Brander and Lewis (1986), and Kim and Maksimovic (1990), the debt level is predetermined or exogenously given at the time the investment decision is made, which is consistent with the assumption above.
Required capital outlay is proportional to the size of investment, $C_0k_0$, where $C_0$ is the unit cost of capital. If the proceeds from the sale of securities in the first place are less than the required capital outlay, the firm could issue more equity at time 0. On the contrary, if the proceeds are larger than the costs of investment, the firm could pay out the difference as a dividend at time 0.

A. Debt Agency Problems and Investment Policy

To demonstrate how debt agency problems influence investment policy, I evaluate the investment decision, $k_0$, first for an all-equity firm, and then for a leveraged firm.

1. All-Equity Firm

I denote the net present value of an all-equity firm at time 0 as $\text{NPV}(k_0)$, where

$$
\text{NPV}(k_0) = \int_{-\infty}^{+\infty} (\mu(k_0) + \sigma(k_0) \epsilon_1) f(\epsilon_1) d\epsilon_1 - C_0k_0
$$

At time 0, an all-equity firm would choose an investment policy, $k_0^*$, to maximize net present value. Then $k_0^*$ must satisfy the following.

$$
\frac{\partial \text{NPV}(k_0)}{\partial k_0} = \mu'(k_0) - C_0 = 0.
$$

Equation (4) suggests that optimal investment is achieved at the point where the marginal return on investment equals its marginal cost. Given that $\mu''(k_0) < 0$, there is a unique value of $k_0^*$ that maximizes the NPV of the firm.

2. Leveraged Firm

In a leveraged firm, the objective of equity holders is to select an investment policy, $k_0^*$, that will maximize the net present value of their equity claims, $\text{ENPV}(k_0)$,

$$
\text{ENPV}(k_0) = \int_{h}^{+\infty} (\mu(k_0) + \sigma(k_0) \epsilon_1 - D) f(\epsilon_1) d\epsilon_1 - C_0k_0,
$$

where

$$
h = \frac{D - \mu(k_0)}{\sigma(k_0)}.
$$

is made. Thus, my study characterizes the effect of leverage on the firm’s subsequent investment decisions. Knowing how the debt level will lead to a suboptimal future investment decision, the firm would choose an ex ante optimal capital structure that ensures equity holders maximize the firm value at time 0.

4The dependence of investment cost on the capital scale allows the model to incorporate the under-investment problem. Green and Talmor (1986) focus only on the risk-shifting problem. In their model, there is no under-investment incentive since they implicitly assume that the cost of investment is independent of scale.
The first-order condition leads to an investment policy in a leveraged firm, \( k_0^d \), which is determined by the following.

\[
\frac{\partial \text{ENPV}(k_0)}{\partial k_0} = \int_{h}^{\infty} \left( \mu'(k_0) + \sigma'(k_0)\epsilon_1 \right) f(\epsilon_1) d\epsilon_1 - C_0 = 0.
\]

To determine how the level of debt in a firm affects equity holders’ investment policy, I totally differentiate equation (6) and get

\[
\frac{\partial k_0^d}{\partial D} = - \frac{\frac{\partial^2 \text{ENPV}(k_0^d)}{\partial k_0^d \partial D}}{\frac{\partial^2 \text{ENPV}(k_0)}{\partial^2 k_0}}.
\]

Assuming that the second-order condition for an interior optimum is satisfied, the denominator on the right-hand side of (7) is negative. Thus, the sign of \( \frac{\partial k_0^d}{\partial D} \) is the same as the sign of the cross-partial derivative in the numerator. Taking derivatives on both sides of equation (6) with respect to \( D \) evaluated at \( k_0^d \), I have

\[
\text{sign} \left( \frac{dk_0^d}{dD} \right) = \text{sign} \left( \frac{\frac{\partial^2 \text{ENPV}(k_0^d)}{\partial k_0^d \partial D}}{\frac{\partial^2 \text{ENPV}(k_0)}{\partial^2 k_0}} \right)
\]

\[
= \text{sign} \left\{ \left( E[\epsilon_1|\epsilon_1 > h] - h \right) \frac{f(h) \sigma'(k_0^d)}{\sigma(k_0^d)} - \frac{f(h)C_0}{\sigma(k_0^d) \int_{h}^{\infty} f(\epsilon_1) d\epsilon_1} \right\}.
\]

A detailed derivation is shown in Appendix A.

The joint effect of the risk-shifting and the under-investment incentives on a firm’s investment policy is clearly illustrated in equation (8). The first term on the right-hand side is related to the risk-shifting incentive, which increases with \( \sigma'(k_0) \). This term contributes to a positive relationship between leverage and investment policy if the marginal volatility of investment, \( \sigma'(k_0) \), is positive, but a negative relationship if \( \sigma'(k_0) \) is negative. The second term is associated with the under-investment incentive, which is proportional to the unit cost of capital, \( C_0 \). This term creates a negative relation between leverage and investment policy. All else equal, the higher is \( C_0 \), the stronger is the under-investment incentive. This is because when \( C_0 \) is high, the increasing debt level, \( D \), will reduce more investment in \( k_0^d \). As a result, the sign of \( (dk_0^d)/(dD) \) will depend on the tradeoff between the two incentive problems.

I first consider that the volatility of project cash flows increases with investment scale, \( \sigma'(k_0^d) > 0 \). As Appendix B proves and Figure 1 depicts, the risk-shifting incentive is dominant at a low level of debt. Increases in leverage will initially lead the firm to over-invest, \( k_0^d > k_0^d \). As the debt level rises, the under-investment incentive becomes more and more significant, and counteracts the risk-shifting problem. Thus, the firm becomes less and less over-invested.

At some high level of debt, the under-investment incentive dominates the firm’s risk-shifting decision, and a leveraged firm under-invests at a level \( k_0^d \) that is less than \( k_0^d \). If the value function of project cash flows is well behaved, there exists a
positive level of debt, $D^*$, at which these two incentives offset each other, and an optimal investment decision is achieved, $k_0^d = k_0$. The optimal level of debt, $D^*$, at which neither an over-investment nor under-investment incentive is induced, can be determined by the following equation,

$$B'(k_0^d) = \mu'(k_0^d)G(h) - \sigma'(k_0^d)\int_{h}^{+\infty} \epsilon f(\epsilon_1) d\epsilon_1 = 0,$$

where $\mu'(k_0^d) = C_0$.

FIGURE 1
The Effect of Leverage on Investment Decision and Debt Agency Cost

Figure 1 shows the effect of leverage on the marginal return of a bond $B'(k_0^d)$, investment decision ($k_0^d$), and the agency cost of debt (AC) when $\sigma'(k_0^d) > 0$. At a low debt level, increases in leverage lead the firm to over-invest, $k_0^d > k_0^d$. The agency cost of debt increases with leverage. As the level of debt increases beyond $D^*$, agency cost declines and reaches zero at $D^*$, at which an optimal investment policy is achieved, $k_0^d = k_0^d$. As the debt level exceeds $D^*$, a leveraged firm under-invests, $k_0^d < k_0^d$, and the agency cost rises with debt level once more.

Alternatively, if the volatility of cash flows declines or does not change with investment scale, $\sigma'(k_0^d) \leq 0$, the risk-shifting incentive encourages a leveraged
firm to under-invest (see Appendix B). Thus, both the risk-shifting and the under-investment problems influence the investment decision in the same way. The higher the debt level, the more the firm under-invests.

B. Agency Cost of Debt

In an efficient capital market, the adverse consequences of debt agency are borne entirely by equity holders through the increased cost of debt financing. Any deviations from firm value-maximizing policies will be reflected in a decline in the value of firm. Following the literature, the agency cost of debt (AC) is defined as the difference between the net present value of an all-equity firm and that of a leveraged firm. Thus,

\[
AC = \mu'(k_0^e) - C_0k_0^e - \mu(k_0^d) + C_0k_0^d,
\]

where \(\mu'(k_0^0) = C_0\). The change in the agency cost corresponding to a change in debt \(D\) is

\[
\frac{\partial AC}{\partial D} = \{C_0 - \mu'(k_0^0)\} \frac{\partial k_0^d}{\partial D}.
\]

Look first at the volatility of project cash flows increasing with investment scale, \(\sigma'(k_0^d) > 0\). As Appendix B proves and Figure 1 depicts, at a low debt level, \((dk_0^d)/(dD) > 0\). Because the risk-shifting incentive is dominant, the firm over-invests, and \(\mu'(k_0^d) < C_0\). Thus, the agency cost of debt rises with the debt level, \(\partial AC/\partial D > 0\). As the level of debt increases beyond a positive level of \(D\), the total agency cost declines with leverage, \(\partial AC/\partial D < 0\). At the optimal debt level, \(D^*\), the two debt agency problems offset each other, and a value-maximizing investment policy is adopted, \(k_0^d = k_0^e\). Thus, the agency cost drops to zero. For a debt level higher than \(D^*\), equity holders' under-investment incentive becomes dominant, and a leveraged firm tends to invest less than the optimal. Hence, \(\partial AC/\partial D > 0\), and the agency cost again becomes positive and rises with leverage.

Figure 1 shows that this dynamic effect of leverage on agency cost is coincident with its influences on investment policy and the marginal return of the bond. Unlike Jensen and Meckling (1976), Myers (1977), and Green and Talmor (1986), my study indicates that, for firms with positive marginal volatility of investment, \(\sigma'(k_0^d) > 0\), the total agency cost of debt does not uniformly increase with the amount of leverage.

Appendix B also shows that firms with zero or negative marginal volatility of investment, \(\sigma'(k_0^d) \leq 0\), experience an under-investment problem at any positive level of debt. Hence, \(\partial AC/\partial D\) is positive, and the agency cost of debt increases monotonically with leverage.

The over-investment problem here arises from equity holders' risk-shifting incentive. This is different from the over-investment incentive discussed by (Stulz (1990) and Li and Li (1996)), which results from the agency costs of managerial discretion given that the managers' perquisites increase with the size of investment.
C. Marginal Volatility of Investment and Debt Policy

The above analysis suggests a countervailing interaction between the two debt agency problems. This potential interaction has not gone unnoticed. Myers (1977) points out that, if investment raises the variance of project return, equity holders' incentive to shift risk will mitigate the under-investment problem. He concludes: "... we may observe risky firms borrowing more than safe ones" (Myers (1977), p. 167). While Myers' argument suggests a positive relationship between the optimal debt level and business risk, the empirical evidence is ambiguous. Long and Malitz (1985) find that a firm's debt level and the riskiness of cash flows are positively related. Bradley, et al. (1984) and Titman and Wessels (1988), however, find a negative and an insignificant relationship, respectively. Empirical tests in Kale, Noe, and Ramirez (1991) support a U-shaped relationship between optimal leverage and business risk.

My simultaneous analysis of the risk-shifting and under-investment problems suggests that it is the marginal volatility of investment (MVI), not the volatility itself, that alleviates the agency problem of under-investment. Although risk-shifting firms may have high volatility in project cash flows, firms with high volatility need not have a high risk-shifting incentive. Simulation analysis in Par- rino and Weisbach (1999) suggests that a firm with high cash flow volatility has a low risk-shifting incentive. This is because, if a firm has assets generating cash flows with high volatility, there are not many additional projects that will increase this volatility so as to shift risk. Thus, I expect a close relationship between the optimal debt level and the marginal volatility, rather than the volatility level.

To investigate this relationship, I assume that the volatility of project cash flow is a linear function of investment scale, \( \sigma(k_D) = ak_D^p + b \).\(^6\) Then marginal volatility, \( \sigma'(k_D^p) \), is equal to the constant, \( a \). I first consider a positive marginal volatility of investment \( (a > 0) \). From equation (9), I can derive \( \partial D^* / \partial a \), and determine its signs.

**Proposition 1.** For firms with positive marginal volatility of investment and high unit cost of capital, \( C_0 \), the optimal debt level increases with the marginal volatility of investment. In contrast, when \( C_0 \) is low, the optimal debt level decreases with the marginal volatility of investment.

**Proof.** See Appendix C.

In this model, the unit cost of capital, \( C_0 \), represents the magnitude of the under-investment problem. As shown above, the higher is \( C_0 \), the more likely a firm is to under-invest. This is because when \( C_0 \) is high, raising the debt level, \( D \), will reduce more investment in \( k_D \). Proposition 1 implies that, for firms with a severe under-investment problem, the risk-shifting incentive alleviates the under-investment problem, thereby reducing the total agency cost of debt financing.

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\( ^6 \) This is the simplest form of a volatility function, which differentiates itself from many previous studies that have only examined the effect of exogenous business risk on corporate capital structure (Castanias (1983), Bradley, Jarrell, and Kim (1984). Kale, Noe, and Ramirez (1991)). It basically decomposes the riskiness of a project cash flow into two parts. One is the endogenous risk, which is proportional to the investment decision, \( k_D \); the other part represents exogenous risk due to market fluctuation that is unrelated to the investment decision.
Thus, such firms can tolerate more debt in their capital structure. For firms with few under-investment problems, however, the dominance of the risk-shifting incentive pushes the optimal capital structure to a corner solution, and the agency cost of debt uniformly rises with debt level. In such a scenario, the optimal debt level would be negatively related to the marginal volatility of investment.

Proposition 1 applies only to firms with positive marginal volatility of investment (MVI). As shown above, if MVI is negative or zero, the risk-shifting incentive leads equity holders to under-invest, and the agency cost of debt increases with the magnitude of MVI. Since the agency cost of debt and the optimal debt level move in opposite directions, the optimal debt level decreases with the magnitude of MVI.

Implication 1. For firms with negative marginal volatility of investment, the optimal debt level decreases with the magnitude of MVI.

III. Empirical Analysis

The above analysis suggests a unique empirical prediction on the relationship of the optimal leverage and the marginal volatility of investment (MVI). This relationship depends on whether the under-investment problem is severe. In the agency literature, it has been well established that the higher is a firm's growth potential, the more likely it is to under-invest. Therefore, I use a measure of growth potential to proxy for the degree of under-investment. Based on Proposition 1 and Implication 1, I propose the following testable hypothesis.

HI. For high growth firms, the optimal level of debt increases with the magnitude of MVI when MVI is positive, but declines with the magnitude of MVI when MVI is negative. It suggests a positive relationship between the optimal debt level and MVI. For low growth firms, the optimal level of debt decreases with the magnitude of MVI for both positive and negative MVI firms. It suggests a negative relationship between the optimal debt level and MVI when MVI is positive, and a positive relationship between the optimal debt level and MVI when MVI is negative.

For high growth firms with a significant under-investment problem, the model suggests that the agency cost of debt will decrease with the magnitude of MVI if the MVI is positive, but increase with the magnitude of MVI if the MVI is negative. This is because the under-investment and risk-shifting incentives offset each other in the former case. For low growth firms where under-investment is likely to be insignificant, the agency cost of debt will increase with the magnitude of MVI for both positive and negative MVI firms, since the risk-shifting problem is the dominant agency problem generated by debt in this case. Since the agency cost of debt and the optimal debt level move in opposite directions, for high growth firms, the optimal debt level will rise with the magnitude of MVI when MVI is positive, but decline with the magnitude of MVI when MVI is negative. For low growth firms, the optimal debt level will decrease with the magnitude of MVI when MVI is positive or negative.
A. Empirical Method

To test the above hypothesis, I first develop an econometric method to estimate the marginal volatility of investment. Second, I test H1 by cross-sectionally relating those estimates of MVI to firms' leverage ratios.

1. Estimation of Marginal Volatility of Investment (MVI)

In a multi-period setting, equation (1) can be rewritten as

\[ \tilde{X}_{t+1} = \mu(k_t) + \sigma(k_t) \varepsilon_{t+1}, \]

where the cash flow in the next period, \( \tilde{X}_{t+1} \), is a random variable whose mean and volatility both depend on current investment \( k_t \). Equation (12) is very general in the sense that the error term, \( \varepsilon_{t+1} \), could be heteroskedastic and autocorrelated.\(^7\) To be consistent with the theoretical model above, I assume a simple linear relationship between the volatility of cash flows and the investment scale,

\[ \sigma(k_t) = ak_t + b, \]

where \( a \) and \( b \) are constants. In addition, I assume a Cobb-Douglas production function for the mean cash flow, \( \mu(k_t) = ck_t^d \), where \( c \) and \( d \) are constants.

I estimate the parameters \((a, b, c, \text{ and } d)\) of this model using the Generalized Method of Moments (GMM) technique. There are several advantages of applying this technique in estimating equation (12). First, the GMM approach does not require that the distribution of cash flow be normal. Second, the GMM estimators and their standard errors are consistent even if the disturbance, \( \varepsilon_{t+1} \), is both heteroskedastic and autocorrelated. Based on equation (12), lagged investment scale \( k_t \) is a natural candidate for an instrumental variable, since it is highly correlated with the true value of investment scale, \( k_t \), but is unlikely to be related to the error term, \( \varepsilon_{t+1} \). As a result, the following population moments implied by the model exactly identify the parameter vector \( \beta = (a, b, c, d) \).

\[ E \left[ \begin{array}{c} \tilde{X}_{t+1} - \mu(k_t) \\ \left( \tilde{X}_{t+1} - \mu(k_t) \right) k_t \\ \left( \tilde{X}_{t+1} - \mu(k_t) \right)^2 - \sigma^2(k_t) \\ \left[ \left( \tilde{X}_{t+1} - \mu(k_t) \right)^2 - \sigma^2(k_t) \right] k_t \end{array} \right] = 0. \]

I use earnings before interest, depreciation, and taxes (EBIDT) as a measure for cash flows, \( \tilde{X}_{t+1} \), and capital expenditure adjusted for depreciation (CAPXAD) for investment scale, \( k_t \).\(^8\) Both EBIDT and CAPXAD are normalized by the book value of assets.

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\(^7\)This would be consistent with product markets, for example, in which good sales this period may likely be followed by good sales next period because the product is superior, or has been marketed successfully.

\(^8\)A firm's real investment expenditures also include advertising and R&D expenses, however, data for advertising and R&D expenses are missing for 80% of the sample. I use an accounting measure of earnings because it is more directly related to investment decisions than stock price. Stock price reflects the present value of all the expected future cash flows, and its volatility would be much noisier than the volatility of cash flows. Moreover, stock prices are directly influenced by the financing decision, which may induce spurious correlation between leverage ratio and MVI in the regression analysis.
Ideally, one would like to obtain estimates of the parameter vector \( \beta \) for individual firms. In the GMM analysis, I estimate four parameters by fitting a nonlinear mean function and a linear volatility function as described above. For nonlinear models, the individual t-test for each parameter is only asymptotically valid. Thus, it requires a large number of data points to get reliable estimates, but the amount of individual firm data is insufficient for this task.

To circumvent this data problem, I pool firms into industry groups based on their SIC codes. This method of grouping is reasonable for three reasons. First, firms in the same industry are likely to have similar cash flow dynamics because they tend to have similar technologies. Second, firms in the same industry are subject to similar economic shocks that may affect the risk-return structure of the investments. Third, investment opportunity sets are similar for firms within the same industry. If a risky project with positive NPV appears in a particular industry, all firms in the industry are able to invest in the project and shift risk. Hence, the marginal volatility of investment is likely similar for firms within the same industry.

2. Other Factors Affecting Financing Policies

The primary focus of the empirical analysis is to examine the model’s prediction on how the optimal capital structure is related to the marginal volatility of investment. It is important, however, to control for competing explanations of debt policy. Omitting other determinants of capital structure from the specification would bias the estimates. In addition, without controlling for those factors, it is possible that the estimates of MVI would not really test the model but instead proxy for other explanatory variables for capital structure, such as a firm’s business risk. Furthermore, including control variables allows determination of the relative contribution of the MVI variable to the firm’s debt policy.

The previous literature provides useful precedents for selecting proxy variables. According to Harris and Raviv (1991), it has been well documented that leverage increases with tangibility of assets and firm size. Leverage decreases with non-debt tax shields, volatility, growth opportunities, and the probability of bankruptcy. I include a number of such variables in the cross-sectional regressions.

B. Data

My data consists of Compustat firms (including those from the research tapes) and covers the years 1976 through 1995. I exclude all financial and real estate firms from the sample (two-digit SIC codes between 60 and 68).

Using the GMM technique, I obtain estimates of marginal volatility of investment, MVI, for different two-digit SIC coded industries from panel data for 1976–1995. To yield reliable estimates without losing too many industries, I select industries in which at least 100 panel data points of both EBIDT and CAPXAD are available for GMM estimation. There are 54 industries satisfying this restriction.

\(^9\)Many firms in the financial services industry do not have information available on earnings before interest, depreciation, and taxes (EBIDT), because such earnings are not meaningful for these companies.
I conduct cross-sectional regression analysis for firms included in the GMM analysis. Independent variables, including firm size (logsize), growth opportunities (market-to-book), tangibility of assets (TANG), non-debt tax shields (ITC), and book leverage ratios, are averages over the 20-year sample period 1976-1995. Earnings volatility (VAR) is calculated as the standard deviation of the first difference in annual EBITDA over the 20-year period divided by the average value of total assets over the same time period. The sample is further restricted to only those firms that have at least three firm-year non-missing time-series values for logsize, market-to-book, TANG, ITC, and book leverage as well as at least three differenced EBITDAs for computing VAR. Table 1 reports the descriptive statistics for all these variables.

**TABLE 1**

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logsize</td>
<td>3452</td>
<td>4.6664</td>
<td>1.9484</td>
<td>-0.0087</td>
<td>10.0712</td>
</tr>
<tr>
<td>Market-to-book</td>
<td>3452</td>
<td>1.4893</td>
<td>0.8122</td>
<td>0.5687</td>
<td>2.4321</td>
</tr>
<tr>
<td>TANG</td>
<td>3452</td>
<td>0.3638</td>
<td>0.2064</td>
<td>0.0133</td>
<td>0.9223</td>
</tr>
<tr>
<td>ITC</td>
<td>3452</td>
<td>0.0336</td>
<td>0.0042</td>
<td>0.0000</td>
<td>0.0369</td>
</tr>
<tr>
<td>VAR</td>
<td>3452</td>
<td>0.0637</td>
<td>0.0716</td>
<td>0.0010</td>
<td>1.1944</td>
</tr>
</tbody>
</table>

Table 1 reports descriptive statistics for firm characteristic variables for a sample of 3452 firms included in the analysis. These firms have at least three firm-year non-missing time-series values for logsize, market-to-book, TANG, ITC, and book leverage as well as at least three differenced EBITDAs for calculating VAR during the sample period 1976-1995. All variables except VAR are averages over the sample period.

Logsize: logarithm of book value of assets.  
TANG: plant and equipment as a percentage of book value of assets.  
ITC: investment tax credits over net sales.  
VAR: standard deviation of the first difference in annual earnings before interest, depreciation, and tax (EBITDA) over the 20-year period divided by the average value of total assets over the same time period.  
Book leverage ratio: ratio of long-term debt over book value of assets.

C. Empirical Results

1. Parameter Estimates from GMM Analysis

There are 54 two-digit SIC industries that meet the sample restrictions, and all parameter estimates converge successfully in the estimation procedure. Table 2 presents descriptive statistics for the GMM estimates. Without any restriction on the estimation procedure, I obtain reasonable estimates of the Cobb-Douglas mean function of cash flows. The coefficients of mean cash flows, c, are positive for all 54 industries, with a mean of 14.6. The exponent coefficients of investment in the mean function, d, are all positive with a minimum of 0.33 and a maximum of 1.12. Furthermore, 52 estimates of d are between zero and one. This fact accords well with the parameter restrictions on the Cobb-Douglas production function, and validates my theoretical assumption that the mean cash flow is an increasing and concave function of investment scale.

For the linear volatility function, the intercept, b, measures the exogenous part of the volatility of cash flows unrelated to investment scale. The finding that
2. Characteristic Differences between Positive and Negative MVI Firms

Given these estimates, I divide firms into two groups. Those from industries with positive marginal volatility of investment are categorized as positive MVI firms, the rest are negative MVI firms. First of all, I compare firm characteristics between the two groups. Table 3 reports the means (in panel A) and medians (in panel B) of the logsize (proxy for firm size), market-to-book (proxy for growth opportunities), ITC (proxy for non-debt tax shields), TANG (proxy for assets tangibility), VAR (proxy for earnings volatility), and book leverage ratios. t-tests and Wilcoxon signed-rank tests are used to examine the difference of means and medians between the two groups respectively.

The logsize and market-to-book ratios of these two groups are not significantly different. Positive MVI firms, however, are systematically different from negative MVI firms in other respects. Positive MVI firms have significantly higher earnings volatility (at the 1% level), but significantly fewer tangible assets and non-debt tax shields (at the 1% level), and significantly lower book leverage ratios (at the 1% level). The lower proportions of tangible assets in positive MVI firms is consistent with the fact that tangible assets can serve as collateral, thereby reducing the firm’s risk-shifting incentive as measured by MVI. Because of the risk-shifting incentive, positive MVI firms have, as expected, higher levels of business risk than negative MVI firms. The differences between the two groups

10 Additional tests on heterogeneity of MVI estimates across firms and industries will be presented and discussed in Section III.C.5.
TABLE 3

Characteristic Differences between Positive and Negative MVI Firms

<table>
<thead>
<tr>
<th>Logsizre</th>
<th>Market-to-Book</th>
<th>ITC</th>
<th>TANG</th>
<th>VAR</th>
<th>Book Leverage Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. Sample Means of Characteristic Variables of Positive and Negative MVI Firms (p-values are reported in parentheses for tests of equality of the two samples based on t-tests assuming unequal variances)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MVI &gt; 0</td>
<td>4.6154</td>
<td>1.4752</td>
<td>0.0034</td>
<td>0.3202</td>
<td>0.0732</td>
</tr>
<tr>
<td>MVI &lt; 0</td>
<td>4.7077</td>
<td>1.6280</td>
<td>0.0042</td>
<td>0.4457</td>
<td>0.0639</td>
</tr>
<tr>
<td>Mean diff.</td>
<td>-0.0923</td>
<td>-0.0528</td>
<td>-0.0008***</td>
<td>-0.1258***</td>
<td>-0.0009***</td>
</tr>
<tr>
<td>(0.1619)</td>
<td>(0.1744)</td>
<td>(0.0001)</td>
<td>(0.0000)</td>
<td>(0.0001)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Panel B. Sample Medians of the Characteristic Variables of Positive and Negative MVI Firms (p-values are reported in parentheses for tests of equality of the medians of the two samples based on Wilcoxon signed-rank tests)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MVI &gt; 0</td>
<td>4.3905</td>
<td>1.2378</td>
<td>0.0022</td>
<td>0.2950</td>
<td>0.0531</td>
</tr>
<tr>
<td>MVI &lt; 0</td>
<td>4.5198</td>
<td>1.1851</td>
<td>0.0030</td>
<td>0.4248</td>
<td>0.0439</td>
</tr>
<tr>
<td>Med. diff.</td>
<td>-0.1293</td>
<td>-0.0525</td>
<td>-0.0008***</td>
<td>-0.1296***</td>
<td>-0.0009***</td>
</tr>
<tr>
<td>(0.1094)</td>
<td>(0.0420)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
</tbody>
</table>

Table 3 reports characteristic differences of samples classified into positive and negative MVI firms according to their marginal volatility of investment (MVI) estimated from GMM analysis. t-tests and Wilcoxon signed-rank tests are used to examine the difference of means and medians between the two groups, respectively.

Logsizre: logarithm of book value of assets.


TANG: plant and equipment as a percentage of book value of assets.

ITC: Investment tax credits over net sales.

VAR: standard deviation of the first difference in annual earnings before interest, depreciation, and tax (EBIDT) over the 20-year period divided by the average value of total assets over the same time period.

Book leverage ratio: ratio of long-term debt over book value of assets.

"••": significant at the 10%, 5%, and 1% levels, respectively.

suggest that it is important to control for these variables in order to examine the relationship between leverage and MVI.

3. Regression Analysis

Hypothesis H1 suggests a differential relationship between leverage and the marginal volatility of investment conditional on firms' growth opportunities. To test H1, I partition the sample into high and low growth firms. I follow Lang, Ofek, and Stulz (1996) and use the growth rate of capital expenditures as a growth measure.11 The three-year growth rate of capital expenditures is defined as the ratio of capital expenditures in year +3 to the capital expenditures in year 0, minus one. Firms are ranked according to their average three-year growth rates of capital expenditures over the sample period. One-third of the firms with the highest growth rate of capital expenditures is categorized as "high growth," and one-third with the lowest growth rate of capital expenditures is categorized as "low growth."12

By construction, the differences in the three-year growth rate of capital expenditures between the high and low growth samples are dramatic. As Table 4 shows, the average three-year growth rate of capital expenditures is 2.6737 for the high growth sample and -0.2366 for the low growth sample, and the differ-

11Following McConnell and Servaes (1995). I avoid using the book-to-market ratio as a growth measure for dividing the sample into high and low growth firms. Sampling on book-to-market ratio before estimating the regression with the same variable violates the assumption of OLS regressions.

12Results of tests using the top and bottom quartiles of the growth classification are more supportive of Hypothesis H1 than the three-group results.
ence is highly significant. I also construct three alternative growth measures of Lang, Ofek, and Stulz (1996); the one-year growth rate of capital expenditures (CAPX1), and one-year and three-year growth rate of employment (EMP1 and EMP3). These measures are all significantly higher for the high growth sample. In contrast, the high growth sample has a significantly lower proportion of tangible assets (TANG), which accords well with the negative relation of asset tangibility and growth opportunities. In addition, the risk-shifting incentive as measured by the MVI is significantly higher for the high growth sample than for the low growth sample. This finding is consistent with the idea that firms with low growth potential and high levels of tangible assets are less likely to shift risk. This is because risk-shifting is more easily monitored by debt holders in these firms.

### TABLE 4

<table>
<thead>
<tr>
<th>Characteristic Differences between High and Low Growth Firms</th>
<th>CAPX3</th>
<th>CAPX1</th>
<th>EMP3</th>
<th>EMP1</th>
<th>TANG</th>
<th>MVI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. Sample Means of Growth Characteristic Variables of High and Low Growth Firms (p-values are reported in parentheses for tests of equality of the means of the two samples based on t-tests assuming unequal variances)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High growth</td>
<td>2.6737</td>
<td>0.6623</td>
<td>1.1253</td>
<td>0.1453</td>
<td>0.3320</td>
<td>0.1749</td>
</tr>
<tr>
<td>Low growth</td>
<td>-0.2386</td>
<td>0.0685</td>
<td>0.0464</td>
<td>0.0190</td>
<td>0.3866</td>
<td>0.1937</td>
</tr>
<tr>
<td>Mean diff.</td>
<td>2.9193***</td>
<td>0.5628***</td>
<td>1.0799***</td>
<td>0.1263***</td>
<td>-0.0578***</td>
<td>0.0362***</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0076)</td>
</tr>
<tr>
<td>Panel B. Sample Medians of Growth Characteristic Variables of High and Low Growth Firms (p-values are reported in parentheses for tests of equality of the medians of the two samples based on Wilcoxon signed-rank tests)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High growth</td>
<td>1.4599</td>
<td>0.4361</td>
<td>0.3558</td>
<td>0.0944</td>
<td>0.2859</td>
<td>0.4565</td>
</tr>
<tr>
<td>Low growth</td>
<td>-0.1650</td>
<td>0.0295</td>
<td>-0.0649</td>
<td>-0.0006</td>
<td>0.3369</td>
<td>0.3305</td>
</tr>
<tr>
<td>Med. diff.</td>
<td>1.6246***</td>
<td>0.4086***</td>
<td>0.4407***</td>
<td>0.1040***</td>
<td>-0.0510***</td>
<td>0.1250**</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0253)</td>
</tr>
</tbody>
</table>

Table 4 reports characteristic differences between samples classified into high and low growth firms according to the following growth measures. t-tests and Wilcoxon signed-rank tests are used to examine the difference of means and medians between the two groups respectively.

- CAPX3: three-year growth rate of capital expenditures, which is the ratio of capital expenditures in year +3 to the capital expenditures in year 0, minus one.
- CAPX1: one-year growth rate of capital expenditures, which is the ratio of capital expenditures in year +1 to the capital expenditures in year 0, minus one.
- EMP3: three-year growth rate of employment, which is the ratio of employment in year +3 to the employment in year 0, minus one.
- EMP1: one-year growth rate of employment, which is the ratio of employment in year +1 to the employment in year 0, minus one.
- TANG: plant and equipment as a percentage of book value of assets.
- MVI: industry estimates of the marginal volatility of investment.

*,**,*** significant at the 10%, 5%, and 1% levels, respectively.

I estimate separate cross-sectional regressions for the high and low growth samples. To control for the characteristic differences between positive and negative MVI firms that are known to explain debt policy, I include logsize, market-to-book, TANG, ITC, and VAR in a regression of the book leverage ratios on estimated MVI. All tests are conducted using a heteroskedasticity-consistent covariance matrix (see White (1980)).

Table 5 reports the results of cross-sectional regressions using the time-series mean of each variable by firm. To identify the relative contribution of the risk-
shifting variable (MVI) to the firm's debt policy, for each sample regressions are carried out without or with the MVI variable. All regressions are highly significant, and have adjusted $R^2$ of about 30%. For high growth firms, the coefficient for logsize is positive and significant. This result suggests that larger firms are more diversified and have lower probability of bankruptcy, thereby adopting more debt in their capital structure. Consistent with the theory of agency cost of debt, the coefficient on the market-to-book ratio is negative and highly significant. In addition, asset tangibility (TANG) has a significant effect with the predicted sign, suggesting that a high fraction of plant and equipment (tangible assets) in the asset base makes high debt level more desirable. However, in contrast to prediction, the coefficient of non-debt tax shields (ITC) is positive and significant at the 1% level.\textsuperscript{14} The coefficient of earnings volatility (VAR) is negative but insignificant. In the regression including MVI, the risk-shifting variable barely changes the coefficients of those control variables. Most important, as the model predicts, the coefficient on MVI is significantly positive. The magnitude of the coefficients indicates that the leverage effect is also economically consequential. The 25th percentile of the MVI is $-1.70$, and the 75th percentile is $1.04$. According to the regression, an increase in MVI from the 25th to the 75th percentile is associated with an increase in leverage ratio of 2%.

For the low growth sample, H1 suggests that the optimal leverage decreases with the absolute value of MVI. To account for the change of the slope term in the linear regression, I estimate the following model,

\begin{equation}
(14) \quad \text{Leverage}_i = \alpha_1 + \alpha_2 D_i + \beta_1 \text{(Logsize)}_i + \beta_2 \text{(Market-to-book)}_i + \beta_3 \text{(TANG)}_i + \beta_4 \text{(ITC)}_i + \beta_5 \text{(VAR)}_i + \beta_6 \text{(MVI)}_i + \beta_7 (D_i \times MVI_i) + \varepsilon_i;
\end{equation}

where $D_i$ is a dummy variable that equals one for negative MVI firms and zero otherwise. $\alpha_2$ and $\beta_7$ represent the difference in intercept and MVI factor loading between negative and positive MVI firms, respectively. The coefficient of MVI for positive MVI firms is $\beta_6$ while the coefficient for negative MVI firms is $\beta_6 + \beta_7$. Table 5 shows that the coefficients of logsize, market-to-book, and TANG are significant and are comparable in magnitude and sign to those observed for the high growth sample. However, ITC has a negative effect on the leverage ratio significant at the level of 5% in the low growth sample.\textsuperscript{15} Interestingly, the coefficient of MVI is negative and significant for positive MVI firms. For negative MVI firms, the coefficient becomes positive and marginally significant at the 10% level. Consistent with H1, this result suggests that for low growth firms, the leverage ratio is negatively related to the absolute value of MVI.

\textsuperscript{14}This is because investment tax credits (ITC) tend to be high when a firm's value depends substantially on tangible assets-in-place. The correlation coefficient of ITC and TANG is 0.27 and significant at the 0.01% level. While the tax theory predicts a negative effect of ITC on debt use, the agency theory predicts a positive coefficient for ITC. Thus, the sign of ITC would depend on the tradeoff between these two opposite effects. A positive coefficient of ITC has been previously documented by Bradley, Jarrell, and Kim (1984) and MacKie-Mason (1990).

\textsuperscript{15}This is because the negative effect of non-debt tax shields on debt policy would be observed only for firms near tax exhaustion so as to affect firms' marginal tax rate. Firms in the low growth sample may be more likely near tax exhaustion than those in the high growth sample.
TABLE 5
Cross-Sectional Regressions Classified by CAPX3

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>High Growth</th>
<th>Low Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.1026***</td>
<td>0.1220***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Logsize</td>
<td>0.0106***</td>
<td>0.0065***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0017)</td>
</tr>
<tr>
<td>Market-to-book</td>
<td>-0.0341***</td>
<td>-0.0268***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>TANG</td>
<td>0.2690***</td>
<td>0.2615***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>ITC</td>
<td>2.9023***</td>
<td>-2.0324**</td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td>(0.0348)</td>
</tr>
<tr>
<td>VAR</td>
<td>-0.0247***</td>
<td>-0.1219**</td>
</tr>
<tr>
<td></td>
<td>(0.8543)</td>
<td>(0.0393)</td>
</tr>
<tr>
<td>MVI</td>
<td>0.0077***</td>
<td>-0.0050**</td>
</tr>
<tr>
<td></td>
<td>(0.0021)</td>
<td>(0.0348)</td>
</tr>
<tr>
<td>D</td>
<td>0.0320</td>
<td>0.0176</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>MVI + D* MVI</td>
<td>0.0048**</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>0.303</td>
<td>0.289</td>
</tr>
<tr>
<td></td>
<td>118</td>
<td>100</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>1172</td>
<td>1172</td>
</tr>
</tbody>
</table>

Table 5 presents cross-sectional analysis of leverage ratio on marginal volatility of investment (MVI) and control variables for samples classified into high and low growth firms according to their three-year growth rate of capital expenditures (CAPX3). P-values, using the heteroskedasticity-consistent standard errors (White (1980)), are in parentheses below the coefficients.

Logsize: logarithm of book value of assets.
TANG: plant and equipment as a percentage of book value of assets.
ITC: investment tax credits over net sales.
VAR: standard deviation of the first difference in annual earnings before interest, depreciation, and tax (EBIDT) over the 20-year period divided by the average value of total assets over the same time period.
D: a dummy variable that equals one for negative MVI firms and zero otherwise.
Book leverage ratio: ratio of long-term debt over book value of assets.
CAPX3: three-year growth rate of capital expenditures, which is the ratio of capital expenditures in year +3 to the capital expenditures in year 0, minus one.

**, ***: significant at the 10%, 5%, and 1% levels, respectively.

4. Tests for Robustness

The empirical results above support the theoretical prediction of the model. However, these results may depend on the specific classification scheme and the variable definitions employed. A particular concern here is whether the three-year growth rate of capital expenditure is a reasonable proxy for the firm's future investment growth opportunities. To shed light on this question, I try alternative measures of growth opportunities, CAPX1, EMP3, and EMP1. Again, I subdivide the sample by top and bottom thirds according to these growth measures, and then estimate regression models separately for the high and low growth firms. The results in Table 6 confirm the earlier findings. For high growth firms, the relation between leverage and MVI is positive and significant; for low growth firms, the relation is significantly negative when MVI is positive and weakly positive when MVI is negative. Furthermore, for both the high and low growth samples, the
coefficients of MVI are comparable in size to those using CAPX3 as a growth measure as in Table 5.

**Table 6**

Cross-Sectional Regressions Classified by CAPX1, EMP3, and EMP1

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>CAPX1</th>
<th>EMP3</th>
<th>EMP1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Growth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>0.0766***</td>
<td>0.1379***</td>
<td>0.1010***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Logsize</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>0.0186***</td>
<td>0.0042</td>
<td>0.0134***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.1032)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Market-to-book</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>-0.0299***</td>
<td>-0.0313***</td>
<td>-0.0354***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>TANG</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>0.3091***</td>
<td>0.2817***</td>
<td>0.2770***</td>
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<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
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<tr>
<td>ITC</td>
<td>High</td>
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<td>High</td>
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<td></td>
<td>1.8424**</td>
<td>-1.0346</td>
<td>2.3493***</td>
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<tr>
<td></td>
<td>(0.0226)</td>
<td>(0.3546)</td>
<td>(0.0007)</td>
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<tr>
<td>VAR</td>
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<td>-0.1546***</td>
<td>-0.0719</td>
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<tr>
<td></td>
<td>(0.4428)</td>
<td>(0.0235)</td>
<td>(0.1465)</td>
</tr>
<tr>
<td>MVI</td>
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</tr>
<tr>
<td></td>
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<td>-0.3057**</td>
<td>0.0057***</td>
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<tr>
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<td>(0.0011)</td>
<td>(0.0433)</td>
<td>(0.0116)</td>
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<td>(0.3786)</td>
<td>(0.1345)</td>
<td>(0.3786)</td>
</tr>
<tr>
<td>MVI+D+MVI</td>
<td>High</td>
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<td>High</td>
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<tr>
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<td>0.0056**</td>
<td>0.0056**</td>
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<td>(0.0698)</td>
<td>(0.0356)</td>
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<td>Adj. R²</td>
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<td>High</td>
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Table 6 presents cross-sectional analysis of leverage ratio on marginal volatility of investment (MVI) and control variables for samples classified into high and low growth firms according to one-year growth rate of capital expenditures (CAPX1), three-year and one-year growth rate of employment (EMP3 and EMP1, respectively). p-values, using the heteroskedasticity-consistent standard errors (White (1980)), are in parentheses below the coefficients.

- Logsize: logarithm of book value of assets.
- TANG: plant and equipment as a percentage of book value of assets.
- ITC: investment tax credits over net sales.
- VAR: standard deviation of the first difference in annual earnings before interest, depreciation, and tax (EBIT) over the 20-year period divided by the average value of total assets over the same time period.
- Book leverage ratio: ratio of long-term debt over book value of assets.
- CAPX1: one-year growth rate of capital expenditures, which is the ratio of capital expenditures in year +1 to the capital expenditures in year 0, minus one.
- EMP3: three-year growth rate of employment, which is the ratio of employment in year +3 to the employment in year 0, minus one.
- EMP1: one-year growth rate of employment, which is the ratio of employment in year +1 to the employment in year 0 minus one.

* *** signficant at the 10%, 5%, and 1% levels, respectively.

I perform an additional sensitivity test by dividing the sample into three groups according to the firm’s price-to-earnings ratio (P/E). The results (not shown) are also consistent with those based upon other classification schemes. Thus, the empirical results appear robust to the choice of a growth measure.
5. Tests for Heterogeneity of GMM Estimates across Firms and Industries

Although I have argued above that firms in the same industry are more likely to have similar MVIIs than firms in different industries, it is important to ensure that there is no significant heterogeneity of MVI across firms in a given industry. I have attempted to address this issue in the following ways.

First, I estimate equation (13) on the firm level, but fail to obtain parameter estimates. For reasons of data limitations, all estimates fail to converge in the estimation procedure. At most, there are only 20 time-series observations for each firm, not enough to get reliable GMM estimates.

Second, I estimate equation (13) for each four-digit SIC industry. There are 240 four-digit SIC industries satisfying my sample restriction criteria, but only 194 of them survive the estimation procedure (parameter estimates of 46 industries fail to converge). This creates a significant problem of sample bias.

Third, I estimate equation (13) for each three-digit SIC industry. There are 18S industries that meet sample restrictions, and 179 of them survive the estimation procedure. I group these parameter estimates according to their two-digit SIC codes, yielding 33 two-digit SIC industries that have more than one estimate at the three-digit level. To measure the heterogeneity of these estimates within a two-digit SIC industry, I compute their standard deviation and range statistics (defined as the maximum minus the minimum) within the same two-digit SIC group. Table 7 presents summary statistics on these 33 variability measures. The average standard deviation of the three-digit estimates of MVI within each two-digit SIC industry is 0.61, about one-third the standard deviation of 1.71 across the two-digit SIC industry (see Table 2). Average standard deviation of the other three parameter estimates within the two-digit SIC industry are one-half or one-third of those across the two-digit SIC industry. The average range statistics of the MVI estimates at the three-digit level is 2.52, whereas the range statistics of MVIIs across the two-digit industry is 7.52. The average range statistics of the other GMM estimates within the two-digit SIC industry are also less than half of those across the two-digit SIC industry. These results suggest that there is much less heterogeneity of MVI estimates within the two-digit SIC industry than across the industry, which justifies the use of the estimates at the two-digit SIC level.

IV. Conclusions

By endogenizing a firm’s risk policy as part of the investment decision, this paper unifies the analysis of the two debt agency problems, risk-shifting and under-investment. A key finding is that, if the volatility of project cash flows increases with investment scale, risk-shifting by equity holders will mitigate the under-investment problem. Thus, contrary to conventional views, the total agency cost of debt is not monotonically increasing with leverage. Furthermore, the study suggests a unique relationship between the optimal debt level and the marginal volatility of investment (MVI), which I define as the change of cash flow volatility corresponding to a change of investment scale. For high growth firms, the optimal level of debt increases with the magnitude of MVI when MVI is positive, but declines with the magnitude of MVI when MVI is negative. For low growth
Table 7 reports summary statistics on the variability of the three-digit SIC Industry estimates within the same two-digit SIC group. Estimates of parameter vector $\beta = (a, b, c, d)$ for each three-digit SIC industry are obtained from GMM analysis. Standard deviation and range statistics of these estimates within each two-digit SIC code group are computed to measure the dispersion of these estimates at the three-digit level. Range statistics is computed as the difference between the maximum and minimum. There are 33 two-digit SIC industries with more than one three-digit estimate so that standard deviation and range statistics can be computed.

Firms, the optimal level of debt decreases with the magnitude of MVI for both positive and negative MVI firms.

I test the model's predictions using cross-sectional data from Compustat (1976–1995). Empirical findings support these predictions. For high growth firms, I find a significant positive relationship between leverage and MVI. For low growth firms, I observe a significantly negative relationship between leverage and MVI when MVI is positive, and a weakly positive relationship between the two when MVI is negative.

Despite these positive empirical results, there are several limitations to this study. First, because of the data limitation, I estimate MVI based on the two-digit SIC industry data instead of firm level data. Although it appears that estimates of MVI are more homogeneous within the same industry than across industries, MVI could be firm specific like the other control variables used in the regressions. Second, because of the dynamic properties of a firm's investment opportunity set, MVI could be time varying instead of constant over the sample period as I have assumed. Third, while I attempt to control for some of the previously identified explanatory variables, it is possible that the estimates of the marginal volatility of investment are proxying for some other unidentified factors that are related to the leverage ratio in the same way as I observed.

Appendix

A. Proof of Equation (8)

Given that the investment policy in a leveraged firm, $k^d_0$, is determined by the following equation,

\[
(A-1) \quad \frac{\partial E(k^d_0)}{\partial k^d_0} = \int_h^{\infty} (\mu' (k^d_0) + \sigma' (k^d_0) \epsilon_1) f(\epsilon_1) d\epsilon_1 - C_0 = 0,
\]

where

\[
h = \frac{D - \mu (k^d_0)}{\sigma (k^d_0)},
\]

Table 7 reports summary statistics on the variability of the three-digit SIC Industry estimates within the same two-digit SIC group. Estimates of parameter vector $\beta = (a, b, c, d)$ for each three-digit SIC industry are obtained from GMM analysis. Standard deviation and range statistics of these estimates within each two-digit SIC code group are computed to measure the dispersion of these estimates at the three-digit level. Range statistics is computed as the difference between the maximum and minimum. There are 33 two-digit SIC industries with more than one three-digit estimate so that standard deviation and range statistics can be computed.
rearranging (A-1) yields

\[
\begin{align*}
\frac{\mu'(k_0^d)}{\sigma'(k_0^d)} &= \frac{\int_{h}^{+\infty} \epsilon_1 f(\epsilon_1) d\epsilon_1}{\int_{h}^{+\infty} f(\epsilon_1) d\epsilon_1} - \frac{C_0}{\sigma'(k_0^d) \int_{h}^{+\infty} f(\epsilon_1) d\epsilon_1} \\
&= \mathbb{E}[\epsilon_1 | \epsilon_1 > h] - \frac{C_0}{\sigma'(k_0^d) \int_{h}^{+\infty} f(\epsilon_1) d\epsilon_1}.
\end{align*}
\]

Taking derivatives with respect to \( k_0^d \) on both sides of equation (A-1), I have

\[
\begin{align*}
\frac{\delta^2 E(k_0^d)}{\partial k_0^d \partial D} &= -(\mu'(k_0^d) + \sigma'(k_0^d) h) \frac{f(h)}{\sigma(k_0^d)} \\
&= (\mathbb{E}[\epsilon_1 | \epsilon_1 > h] - h) \frac{f(h) \sigma'(k_0^d)}{\sigma(k_0^d)} \\
&\quad - \frac{f(h)C_0}{\sigma(k_0^d) \int_{h}^{+\infty} f(\epsilon_1) d\epsilon_1}.
\end{align*}
\]

The second equality follows from equation (A-2).

B. Proof of the Dynamic Effect of Leverage (D) on a Firm's Investment Policy (\( k_0^d \))

To dissect the effect of leverage on a firm's investment policy, I define the total value of debt at time 0 as \( B(k_0^d) \),

\[
B(k_0^d) = \int_{h}^{+\infty} D f(\epsilon_1) d\epsilon_1 + \int_{\epsilon_1^*}^{h} (\mu(k_0^d) + \sigma(k_0^d) \epsilon_1) f(\epsilon_1) d\epsilon_1.
\]

The change in the bond value consequent to a change in \( k_0 \) is

\[
B'(k_0^d) = \mu'(k_0^d) F(h) - \sigma'(k_0^d) \int_{h}^{+\infty} \epsilon_1 f(\epsilon_1) d\epsilon_1,
\]

where \( F(\cdot) \) is the cumulative distribution function of \( \epsilon_1 \). Thus,

\[
\begin{align*}
\frac{\delta^2 B(k_0^d)}{\partial k_0^d \partial D} &= (\mu'(k_0^d) + \sigma'(k_0^d) h) \frac{f(h)}{\sigma(k_0^d)} \\
&= - \left\{ \mathbb{E}[\epsilon_1 | \epsilon_1 > h] - h - \frac{C_0}{\sigma'(k_0^d) \int_{h}^{+\infty} f(\epsilon_1) d\epsilon_1} \right\} \\
&\quad \times \frac{f(h) \sigma'(k_0^d)}{\sigma(k_0^d)} \\
&= \left\{ C_0 + \sigma'(k_0^d) h \int_{h}^{+\infty} f(\epsilon_1) d\epsilon_1 - \sigma'(k_0^d) \int_{h}^{+\infty} \epsilon_1 f(\epsilon_1) d\epsilon_1 \right\} \\
&\quad \times \frac{f(h)}{\sigma(k_0^d) \int_{h}^{+\infty} f(\epsilon_1) d\epsilon_1}.
\end{align*}
\]
Define $X = C_0 + \sigma'(k_0^d) \int_{h}^{+\infty} f(\xi_1) d\xi_1 - \sigma'(k_0^d) \int_{h}^{+\infty} \xi_1 f(\xi_1) d\xi_1.$

Case 1. $\sigma'(k_0) > 0.$

\[
\frac{\partial X}{\partial D} = \frac{\sigma'(k_0^d) \int_{h}^{+\infty} f(\xi_1) d\xi_1}{\sigma(k_0^d)} - \frac{\sigma'(k_0^d) h f(h)}{\sigma(k_0^d)} + \frac{\sigma'(k_0^d) h f(h)}{\sigma(k_0^d)} \\
= \frac{\sigma'(k_0^d) \int_{h}^{+\infty} f(\xi_1) d\xi_1}{\sigma(k_0^d)} > 0.
\]

So $X$ is a monotonically increasing function of $D$. As $D = 0$, $\mu'(k_0^d) = C_0$, and $B'(k_0^d) = 0$.

At a low level of debt, $h = (D - \mu(k_0^d))/\sigma(k_0^d)$ is negative and very small, then $X < 0$. Based on (B-3), $(\partial^2 B(k_0^d)/(\partial k_0^d \partial D)) < 0$. Hence, $B'(k_0^d) < 0$, and $k_0^d > k_0^d$. This suggests that a levered firm over-invests.

As the debt level increases, $h = (D - \mu(k_0^d))/\sigma(k_0^d)$ becomes larger and even positive. Since $X$ is a uniformly rising function of $D$, there exists a level of debt, $D'$, at which $X = 0$. $D'$ is determined by the following equation,

(B-4) $C_0 + \sigma'(k_0^d) \int_{h}^{+\infty} f(\xi_1) d\xi_1 - \sigma'(k_0^d) \int_{h}^{+\infty} \xi_1 f(\xi_1) d\xi_1 = 0.$

At this debt level, $D'$, $(\partial^2 B(k_0^d))/(\partial k_0^d \partial D) = 0$. Then as debt increases more,

\[
C_0 + \sigma'(k_0^d) \int_{h}^{+\infty} f(\xi_1) d\xi_1 - \sigma'(k_0^d) \int_{h}^{+\infty} \xi_1 f(\xi_1) d\xi_1 > 0,
\]

$(\partial^2 B(k_0^d))/(\partial k_0^d \partial D)$ switches sign and becomes positive. If the function of $B'(k_0^d)$ is well behaved, there exists a positive debt level $D^*$, at which $B'(k_0^d) = 0$, and $k_0^d = k_0^d$. It implies that, in the presence of debt level $D^*$, a levered firm would adopt an optimal investment policy as an all-equity firm. And $D^*$ is determined by

(B-5) $\mu'(k_0^d) F(h) - \sigma'(k_0^d) \int_{h}^{+\infty} \xi_1 f(\xi_1) d\xi_1 = 0,$

where

\[
\mu'(k_0^d) = C_0.
\]

When the level of debt is even higher than $D^*$, since $(\partial^2 B(k_0^d))/(\partial k_0^d \partial D) > 0$, then $B'(k_0^d) > 0$, and $k_0^d < k_0^d$, indicating that a leveraged firm under-invests.

Case 2. $\sigma'(k_0) \leq 0.$

As shown above in (B-3),

\[
\frac{\partial^2 B(k_0^d)}{\partial k_0^d \partial D} = - \left\{ \frac{E[\xi_1 | \xi_1 > h] - h}{\sigma'(k_0^d) \int_{h}^{+\infty} f(\xi_1) d\xi_1} \right\} \frac{f(h) \sigma'(k_0^d)}{\sigma(k_0^d)}.
\]

Since $E[\xi_1 | \xi_1 > h] - h > 0$, it is always true that $(\partial^2 B(k_0^d))/(\partial k_0^d \partial D) > 0$. As $D = 0$, $B'(k_0^d) = 0$. For any positive level of debt $D$, $B'(k_0^d) > 0$, and then $k_0^d < k_0^d$. $B'_0(k_0^d)$ is uniformly increasing with $D$. 


C. Proof of Proposition 1

Assuming that \( \sigma(k^d_0) = ak^d_0 + b \), the marginal volatility of investment \( \sigma'(k^d_0) = a \). Since the optimal debt level, \( D^* \) must satisfy the following equation,

\[
(C-1) \quad B'(k^d_0) = \mu'(k^d_0) F(h) - \sigma'(k^d_0) \int_{h}^{+\infty} \epsilon_1 f(\epsilon_1) d\epsilon_1 = 0,
\]

where \( \mu'(k^d_0) = C_0 \), and \( h = (D - \mu(k^d_0)) / (\sigma(k^d_0)) \), in order to determine the relation between \( D^* \) and the marginal volatility of investment \( a \), I totally differentiate (C-1) with respect to \( a \), which yields

\[
\frac{\partial D^*}{\partial a} = - \frac{\frac{\partial^2 B(k^d_0)}{\partial k^d_0 \partial a}}{\frac{\partial^2 B(k^d_0)}{\partial k^d_0 ^2 \partial D^*}}.
\]

Given that \( \frac{\partial^2 B(k^d_0)}{\partial k^d_0 ^2 \partial D} > 0 \) at \( D^* \),

\[
\text{sign} \left( \frac{dD^*}{da} \right) = \text{sign} \left( - \frac{\frac{\partial^2 B(k^d_0)}{\partial k^d_0 ^2 \partial a}}{\frac{\partial^2 B(k^d_0)}{\partial k^d_0 ^2 \partial D^*}} \right)
\]

\[
= \text{sign} \left\{ -C_0 f(h) \frac{\partial h}{\partial a} - ahf(h) \frac{\partial h}{\partial a} + \int_{h}^{+\infty} \epsilon_1 f(\epsilon_1) d\epsilon_1 \right\}.
\]

Since \( \frac{\partial h}{\partial a} = \frac{(D^* - \mu(k^d_0)) k^d_0}{(ak^d_0 + b)^2} \),

I have

\[
(C-2) \quad \text{sign} \left( \frac{dD^*}{da} \right) = \text{sign} \left\{ (C_0 + ah)f(h) \frac{hk^d_0}{ak^d_0 + b} + \int_{h}^{+\infty} \epsilon_1 f(\epsilon_1) d\epsilon_1 \right\}.
\]

As shown in Appendix subsection A, at the optimal debt level \( D^* \), \( \frac{\partial^2 B(k^d_0)}{\partial k^d_0 ^2 \partial D} > 0 \). Thus,

\[
\frac{\partial^2 B(k^d_0)}{\partial k^d_0 ^2 \partial D} = \left( \mu'(k^d_0) + \sigma'(k^d_0) h \right) \frac{f(h)}{\sigma(k^d_0)}
\]

\[
= \frac{(C_0 + ah)f(h)}{ak^d_0 + b} > 0.
\]

In addition, \( \int_{h}^{+\infty} \epsilon_1 f(\epsilon_1) d\epsilon_1 > 0 \), thus the sign of \( \partial D^* / \partial a \) will depend on the sign of \( h = (D - \mu(k^d_0)) / (\sigma(k^d_0)) \), which is dependent on the optimal investment scale, \( k^d_0 \), which is in turn related to the unit cost of capital, \( C_0 \). Given that \( \mu'(k) = C_0, \partial k/\partial C_0 = 1/\mu''(k) < 0 \). Hence, \( k^d_0 \) is a decreasing function of \( C_0 \). When \( C_0 \) is high, \( k^d_0 \) will be low. Since \( \mu(k^d_0) \) is an increasing function of \( k^d_0 \), \( \mu(k^d_0) \) would be low, and then \( h = (D - \mu(k^d_0)) / (\sigma(k^d_0)) \) is positive. Therefore \( \partial D^* / \partial a > 0 \). In contrast, when \( C_0 \) is low, \( k^d_0 \) and \( \mu(k^d_0) \) will be high, so \( h < 0 \). Thus, a negative \( \partial D^* / \partial a \) yields.
References


